

Illustrative Mathematics

5.NF Reasoning about Multiplication

Alignments to Content Standards

- [Alignment: 5.NF.B.5](#)

Tags

- *This task is not yet tagged.*

Your classmate Ellen says,

When you multiply by a number, you will always get a bigger answer. Look, I can show you.

Start with 9.

Multiply by 5. $9 \times 5 = 45$

The answer is 45, and $45 > 9$

45 is bigger than 9.

It even works for fractions.

Start with $\frac{1}{2}$.

Multiply by 4. $\frac{1}{2} \times 4 = 2$

The answer is 2, and $2 > \frac{1}{2}$

2 is bigger than $\frac{1}{2}$.

Ellen's calculations are correct, but her rule does not always work.

For what numbers will Ellen's rule work? For what numbers will Ellen's rule not work? Explain and give examples.

Commentary

The solution shown below is in adult language. Responses by fifth grade students will vary widely in their level of precision. This is a good task to work with kids to try to explain their thinking clearly and precisely, although teachers should be willing to work with many different ways of explaining the relationship between the magnitude of the factors and the magnitude of the product.

This relationship can be expressed using symbolic notation more succinctly. Assuming that $x > 0$, then:

$$x \cdot a < x \quad \text{if} \quad 0 < a < 1$$

and

$$x \cdot a > x \quad \text{if} \quad a > 1.$$

It is worth noting here that the intuitions about multiplication represented by this task and standard will be very useful in algebra courses and beyond, especially when students study transformations.

Note that the situation requires additional thought when you multiply signed numbers. However, students will not deal with products of signed numbers until seventh grade (see 7.NS.2).

Solutions

Solution: Thinking of multiplication as scaling

Whenever you multiply a positive number by a factor greater than 1, the product will be larger than the original number. Both of Ellen's choices illustrate this principle.

Whenever you multiply a positive number by a positive factor less than 1, the product will be smaller than the original number. For example,

$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}.$$

Both factors are less than 1, and the product is less than both factors.

Of course, whenever you multiply a number by 1, the product will be equal to the original number.



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