**Overview:** *The overview statement is intended to provide a summary of major themes in this unit (as adapted from The Progressions for the Common Core State Standards in Mathematics).*

In this unit, students focus on reasoning about number or shape patterns, connecting a rule for a given pattern with its sequence of numbers or shapes. Patterns that consist of repeated sequences of shapes or growing sequences of designs can be appropriate for instruction. For example, students could examine a sequence of dot designs in which each design has 4 more dots than the previous one and they could reason about how the dots are organized in the design to determine the total number of dots in the 100th design. In examining numerical sequences, fourth graders can explore rules of repeatedly adding the same whole number or repeatedly multiplying by the same whole number. Properties of repeating patterns of shapes can be explored with division. For example, to determine the 100th shape in a pattern that consists of repetitions of the sequence “square, circle, triangle,” the fact that when we divide 100 by 3 the whole number quotient is 33 with a remainder 1 tells us that after 33 full repeats, the 99th shape will be a triangle (the last shape in the repeating pattern), so the 100th shape is the first shape in the pattern, which is a square. Notice that the Standards do not require students to infer or guess the underlying rule for a pattern, but rather ask them to generate a pattern from a given rule and identify features of the given rule.

**Teacher Notes:** *The information in this component provides additional insights which will help the educator in the planning process for the unit.*

* Review the Progressions for K, Counting and Cardinality; K-5 Operations & Algebraic Thinking at: <http://commoncoretools.me/2011/05/29/complete-draft-progression-for-cc-and-oa/> to see the development of the understanding of generating and analyzing patterns as stated by the Common Core Standards Writing Team, which is also the guiding information for the PARCC Assessment development.
* When implementing this unit, be sure to incorporate the Enduring Understandings and Essential Questions as the foundation for your instruction, as appropriate.
* Students should engage in well-chosen, purposeful, problem-based tasks. A good mathematics problem can be defined as any task or activity for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific correct solution method (Hiebert et al., 1997). A good mathematics problem will have multiple entry points and require students to make sense of the mathematics. It should also foster the development of efficient computations strategies as well as require justifications or explanations for answers and methods.
* Instruction should include work with both numeric patterns and shape patterns.
* It is important to allow time during instruction for students to share what they notice about the pattern being explored. This often helps develop an understanding of numeric relationships. For example, if the rule is to multiply a number by 2 and then add 1, the result will always be an odd number. Discussing and understanding this can lead to more accuracy during computation.
* Instruction should include work with both repeating and growing patterns.

**Enduring Understandings:** *Enduring understandings**go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.*

* Patterns enable us to discover, analyze, describe, extend, and formulate concrete understandings of mathematical and real world phenomena.
* Algebraic representations are used to communicate and generalize patterns in mathematics.
* Changes in quantities can be used to predict outcomes and solve problems.
* Patterns can be found in many forms.
* Patterns can grow and repeat.
* Patterns can be generalized.
* Mathematical expressions represent relationships.
* Number patterns and relationships can be represented using variables.

**Essential Questions:** *A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.*

* How can patterns help in problem solving?
* How can multiple representations be used to express relationships?
* How can expressions and equations be used to represent practical problems symbolically?
* How can change be described mathematically?
* Why are equations and inequalities useful?
* What can we learn from studying patterns?
* How can we use patterns to develop an understanding of our place value system?
* What patterns are useful in computation?
* Where are patterns in nature, architecture, music, words, and numbers?
* What is the repeating and/or increasing unit in the pattern?
* What strategies can be used to continue a sequence?
* How does finding patterns help in counting and/or computation?
* How is an equation like a balance scale?
* How can relationships be expressed symbolically?
* Why are variables used?
* What strategies can be used to solve for unknowns?

**Content Emphasis by Cluster in Grade 4:** *According to the Partnership for the Assessment of Readiness for College and Careers (PARCC), some clusters require greater emphasis than others. The table below shows PARCC’s relative emphasis for each cluster. Prioritization does not imply neglect or exclusion of material. Clear priorities are intended to ensure that the relative importance of content is properly attended to. Note that the prioritization is in terms of cluster headings.*

**Key:**

* ***Major Clusters***
* ***Supporting Clusters***
* ***Additional Clusters***

Operations and Algebraic Thinking

* Use the four operations with whole numbers to solve problems.
* Gain familiarity with factors and multiples.
* ***Generate and analyze patterns.***

Number and operations in Base Ten

* Generalize place value understanding for multi-digit whole numbers.
* Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations – Fractions

* Extend understanding of fraction equivalence and ordering.
* Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
* Understand decimal notation for fractions, and compare decimal fractions.

Measurement and Data

* Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
* Represent and interpret data.
* Geometric measurement: understand concepts of angle and measure angles.

Geometry

* Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

**Focus Standards:** *(Listed as Examples of Opportunities for In-Depth Focus in the PARCC Content Framework document):*

*According to the Partnership for the Assessment of Readiness for College and Careers (PARCC), this component highlights some individual standards that play an important role in the content of this unit. Educators should give the indicated mathematics an especially in-depth treatment, as measured for example by the number of days; the quality of classroom activities for exploration and reasoning; the amount of student practice; and the rigor of expectations for depth of understanding or mastery of skills.*

* **4.NBT.B.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
* **4.NBT.B.6** Find whole-number quotients and remainders, with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

**Possible Student Outcomes:** *The following list provides outcomes that describe the knowledge and skills that students should understand and be able to do when the unit is completed. The outcomes are often components of more broadly-worded standards and sometimes address knowledge and skills necessarily related to the standards. The lists of outcomes are not exhaustive, and the outcomes should not supplant the standards themselves. Rather, they are designed to help teachers delve deeply into the standards and augment as necessary, providing added focus and clarity for lesson planning purposes. This list is not intended to imply any particular scope or sequence.*

The student will:

* Generate a number pattern that follows a given rule.
* Generate a shape pattern that follows a given rule.
* Generate both growing and repeating patterns that follow given rules.
* Identify various features that they see in the patterns generated.
* Determine a future element in a repeating or growing pattern based on the portion of the pattern visible.

**Progressions from Common Core State Standards in Mathematics:** *For an in-depth discussion of the overarching, “big picture” perspective on student learning of content related to this unit, see:*

* The Common Core Standards Writing Team (29 May 2011), Progressions for K, Counting and Cardinality; K-5 Operations & Algebraic Thinking at: <http://commoncoretools.me/2011/05/29/complete-draft-progression-for-cc-and-oa/>

**Vertical Alignment:** *Vertical curriculum alignment provides two pieces of information: (1) a description of prior learning that should support the learning of the concepts in this unit, and (2) a description of how the concepts studied in this unit will support the learning of additional mathematics.*

* **Key Advances from Previous Grades:**

Students in Grade 2:

* Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s.
* Count within 1000; skip-count by 5s, 10s, and 100s.

Students in Grade 3:

* Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.
* **Additional Mathematics:**

Students in Grade 5:

* Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms.
* Form ordered pairs consisting of corresponding terms from the two patterns, and graph the order pairs on a coordinate plane.
* Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
* Explain patterns in the number of zeros of the product when multiplying a number by powers of 10.
* Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.

Students in Grade 8:

* Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
* Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from tow (x, y) values, including reading these from a table or from a graph.
* Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
* Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear).
* Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
* Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
* Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
* Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
* Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated from rows or columns to describe possible association between the two variables.

**Possible Organization of Unit Standards:** *This table identifies additional grade-level standards within a given cluster that support the over-arching unit standards from within the same cluster. The table also provides instructional connections to grade-level standards from outside the cluster.*

|  |  |  |
| --- | --- | --- |
| **Over-Arching**  **Standards** | **Supporting Standards**  **within the Cluster** | **Instructional Connections outside the Cluster** |
| **4.OA.C.5:** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* |  | **4.OA.B.4:** Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range of 1-100 is prime or composite.  **4.NBT.A.1:** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* |

**Connections to the Standards for Mathematical Practice:** *This section provides examples of learning experiences for this unit that support the development of the proficiencies described in the Standards for Mathematical Practice. These proficiencies correspond to those developed through the Literacy Standards. The statements provided offer a few examples of connections between the Standards for Mathematical Practice and the Content Standards of this unit. The list is not exhaustive and will hopefully prompt further reflection and discussion.*

***In this unit, educators should consider implementing learning experiences which provide opportunities for students to:***

1. **Make sense of problems and persevere in solving them.**
   1. Determine what the problem is asking for: pattern generation, missing element in a pattern, comparison of two patterns, apparent features of the pattern that are not explicit in the rule.
   2. Determine whether concrete or virtual models, pictures, mental mathematics, tables, or equations are the best tools for solving the problem.
   3. Check the solution with the problem to verify that it does answer the question asked.
2. **Reason abstractly and quantitatively**
   1. Compare the your reasoning about the pattern with that of your classmates.
   2. Use the attributes of the pattern to justify your thinking.
   3. Use the elements of the pattern to verify the rule.
3. **Construct Viable Arguments and critique the reasoning of others.**
   1. Compare the sequences or models used by others with yours.
   2. Examine the steps taken that produce an incorrect response and provide a viable argument as to why the process produced an incorrect response.
   3. Use the calculator to verify the correct numeric pattern, when appropriate.
4. **Model with Mathematics**
   1. Construct visual models using concrete or virtual manipulatives, pictures, numbers, or equations to justify thinking and display the pattern or solution.
5. **Use appropriate tools strategically**
   1. Use base ten manipulative models, counters, addition or multiplication tables, pattern blocks, or other models, as appropriate.
   2. Use the calculator to verify computation.
6. **Attend to precision**
   1. Use mathematics vocabulary such as repeating pattern, growing pattern, element, addend, product, multiple, factor, etc. properly when discussing patterns.
   2. Demonstrate understanding of the mathematical processes required to solve a problem by carefully showing all of the steps in the solving process.
   3. Correctly write or represent patterns and/or equations.
   4. Use <, =, and > appropriately to compare expressions.
7. **Look for and make use of structure.**
8. Use the relationship between the elements of the pattern to explain the rule.
9. Use the relationships demonstrated in the pattern to discuss the attributes of the pattern.
10. Compare two patterns, stating their similarities and differences.
11. **Look for and express regularity in reasoning**
    1. Use the patterns illustrated to justify the rule stated.
    2. Use the relationships between the elements in the pattern to verify the rule.

**Content Standards with Essential Skills and Knowledge Statements and Clarifications:** *The Content Standards and Essential Skills and Knowledge statements shown in this section come directly from the Maryland State Common Core Curriculum Frameworks. Clarifications were added as needed. Educators should be cautioned against perceiving this as a checklist. All information added is intended to help the reader gain a better understanding of the standards.*

| **Standard** | **Essential Skills and Knowledge** | **Clarification** |
| --- | --- | --- |
| **4.OA.C.5**  Generate a number or shape pattern that follows a given rule. Identify **apparent features** of the **pattern** that were not explicit in the rule itself. *For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* | **Essential Skills and Knowledge**   * **Ability to apply knowledge of Growing Patterns versus Repeating Patterns using either numbers or shapes** | Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations.  Every pattern is based on a rule. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will be. Students investigate different patterns to identify apparent features of the pattern and to justify why those features exist.  Examples:   |  |  |  | | --- | --- | --- | | Pattern | Rule | Feature(s) | | 2, 7, 12, 17, 22, 27, 32, 37… | Start with 2, add 5 | The numbers alternately end with a 2 or 7, moving from even to odd. | | 8, 32, 128, 512, 2048 … | Start with 2, multiply by 4 | The numbers are multiples of 4 and end with either 8 or2. The numbers are all even numbers since both factors are even. |   After students have applied rules to generate a pattern, they need to discuss the different features or attributes that they notice in the pattern, as modeled in the chart.  *(Adapted from the Arizona State Curriculum site at*  *<http://www.azed.gov/standards-practices/mathematics-standards/>.)* |

**Evidence of Student Learning:** *The Partnership for the Assessment of Readiness for College and Careers (PARCC) has awarded the Dana Center a grant to develop the information for this component. This information will be provided at a later date. The Dana Center, located at the University of Texas in Austin, encourages high academic standards in mathematics by working in partnership with local, state, and national education entities.  Educators at the Center collaborate with their partners to help school systems nurture students' intellectual passions.  The Center advocates for every student leaving school prepared for success in postsecondary education and in the contemporary workplace.*

**Fluency Expectations and Examples of Culminating Standards:** *This section highlights individual standards that set expectations for fluency, or that otherwise represent culminating masteries. These standards highlight the need to provide sufficient supports and opportunities for practice to help students meet these expectations. Fluency is not meant to come at the expense of understanding, but is an outcome of a progression of learning and sufficient thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend one or more grades earlier in the standards than the grade when fluency is finally expected.*

* Students fluently add and subtract multi-digit whole numbers using the standard algorithm.

**Common Misconceptions:** *This list includes general misunderstandings and issues that frequently hinder student mastery of concepts regarding the content of this unit.*

* Make an error when skip-counting when extending a pattern.
* Thinking that only repeating patterns are patterns.
* Thinking that a pattern only involves two elements.
* Repeating a given element instead of increasing or decreasing the pattern based on the rule.
* Assuming that patterns are only made with shapes, not numbers.

**Interdisciplinary Connections:** *Interdisciplinary connections fall into a number of related categories:*

* *Literacy standards within the Maryland Common Core State Curriculum*
* *Science, Technology, Engineering, and Mathematics standards*
* *Instructional connections to mathematics that will be established by local school systems, and will reflect their specific grade-level coursework in other content areas, such as English language arts, reading, science, social studies, world languages, physical education, and fine arts, among others.*

|  |  |  |
| --- | --- | --- |
| **Available Model Lesson Plan(s)** | | |
| The lesson plan(s) have been written with specific standards in mind.  Each model lesson plan is only a MODEL – one way the lesson could be developed.  We have NOT included any references to the timing associated with delivering this model.  Each teacher will need to make decisions related to the timing of the lesson plan based on the learning needs of students in the class. The model lesson plans are designed to generate evidence of student understanding.  This chart indicates one or more lesson plans which have been developed for this unit. Lesson plans are being written and posted on the Curriculum Management System as they are completed. Please check back periodically for additional postings. | | |
| **Standards Addressed** | **Title** | **Description/Suggested Use** |
| 4.OA.C.5 | Numeric & Shape Patterns | Students explore both numeric and shape patterns in the lesson. They use the multiplication chart, a function table, manipulatives, and drawings to make sense of the patterns, extend them, and solve problems. |

|  |  |  |
| --- | --- | --- |
| **Available Lesson Seeds** | | |
| The lesson seed(s) have been written with specific standards in mind.  These suggested activity/activities are not intended to be prescriptive, exhaustive, or sequential; they simply demonstrate how specific content can be used to help students learn the skills described in the standards. Seeds are designed to give teachers ideas for developing their own activities in order to generate evidence of student understanding.  This chart indicates one or more lesson seeds which have been developed for this unit. Lesson seeds are being written and posted on the Curriculum Management System as they are completed. Please check back periodically for additional postings. | | |
| **Standards Addressed** | **Title** | **Description/Suggested Use** |
| 4.OA.C.5 | Pattern Matching Game | Students play a game in which they match a numeric or shape pattern with its rule. |
|  |  |  |

**Sample Assessment Items:** *The items included in this component will be aligned to the standards in the unit and will include:*

* + *Items purchased from vendors*
  + *PARCC prototype items*
  + *PARCC public released items*
  + *Maryland Public release items*
  + *Formative Assessment*

|  |  |  |  |
| --- | --- | --- | --- |
| **Topic** | **Standards Addressed** | **Link** | **Notes** |
| Generate and analyze patterns |  | <http://www.illustrativemathematics.org/standards/k8> | See the Illustrative Mathematics site. This task is meant to be used in an instructional setting. Part (b) is intentionally left open-ended to encourage students to develop the habit of looking for patterns that might hint at some underlying structure as described in Standard for Mathematical Practice 7. |
| Generate and analyze patterns |  | <http://www.k-5mathteachingresources.com/support-files/square-numbers.pdf>  <http://www.k-5mathteachingresources.com/support-files/triangular-numbers.pdf> | See the K-5 Math Teaching Resources site for two growing pattern problems that takes them beyond just generating a pattern, to analyzing it as well. |
| Generate and analyze patterns |  | <https://www.georgiastandards.org/Common-Core/Common%20Core%20Frameworks/CCGPS_Math_4_Unit1FrameworkSE.pdf> | See the Georgia Standards site. See pages 90-92 for a pattern lesson using the hundred chart.  See pages 97-103 for a growing pattern lesson related to Earth Day. |

**Interventions/Enrichments:** *(Standard-specific modules that focus on student interventions/enrichments and on professional development for teachers will be included later, as available from the vendor(s) producing the modules.)*

**Vocabulary/Terminology/Concepts:** *This section of the Unit Plan is divided into two parts. Part I contains vocabulary and terminology from standards that comprise the cluster which is the focus of this unit plan. Part II contains vocabulary and terminology from standards outside of the focus cluster. These “outside standards” provide important instructional connections to the focus cluster.*

***Part I – Focus Cluster:***

***apparent features of a pattern:***  identifying features of a pattern that are not explicit in the rule itself. For example, when skip counting by 5 beginning with 5, the ones digit is always a 5 or 0. However, when skip counting by 5 beginning with 3, the ones digit is always a 3 or 8.

***pattern:***  a regularity in a situation such as those in nature as in spirals on pineapples; shapes as in geometric designs in quilts; number sequences as in 3, 6, 9, 12…

***growing pattern:*** a pattern that involves a progression from step to step.

Example:

***repeating pattern:*** a pattern in which a core is repeated.

Example:

***Part II – Instructional Connections outside the Focus Cluster***

***multiple:*** is the product of a whole number and any other whole number. Example: 20 is a multiple of 5 because 4 × 5 = 20.

***prime:*** is a whole number greater than 1 that has exactly two factors, 1 and the number itself.

***composite:*** is a whole number that has more than two factors.

***base ten numerals:*** a base of a numeration system is the number that is raised to various powers to generate the place values of that system. In the base ten numeration system the base is ten. The first place is 100 or 1 (the units place), the second is 101 or 10 (the tens place, the third is 102 or 100 (the hundreds place), etc. This determines the place value of the different positions in a number. Example: One number written as a base ten numeral would be *6,427*.

**Resources:**

**Free Resources:**

* <http://wps.ablongman.com/ab_vandewalle_math_6/0,12312,3547876-,00.html> Reproducible blackline masters
* <http://lrt.ednet.ns.ca/PD/BLM_Ess11/table_of_contents.htm> mathematics blackline masters
* <http://yourtherapysource.com/freestuff.html> Simple activities to encourage physical activity in the classroom
* <http://www.mathsolutions.com/index.cfm?page=wp9&crid=56> Free lesson plan ideas for different grade levels
* <http://sci.tamucc.edu/~eyoung/literature.html> links to mathematics-related children’s literature
* <http://www.nctm.org/> National Council of Teachers of Mathematics
* [www.k-5mathteachingresources.com](http://www.k-5mathteachingresources.com) Extensive collection of free resources, math games, and hands-on math activities aligned with the Common Core State Standards for Mathematics
* <http://elementarymath.cmswiki.wikispaces.net/Standards+for+Mathematical+Practice> Common Core Mathematical Practices in Spanish
* <http://mathwire.com/> Mathematics games, activities, and resources for different grade levels
* <http://www.pbs.org/teachers/math/> interactive online and offline lesson plans to engage students. Database is searchable by grade level and content
* <http://www.k8accesscenter.org/training_resources/MathWebResources.asp> valuable resource including a large annotated list of free web-based math tools and activities.
* <http://www.cast.org/udl/index.html> Universal Design for Learning
* <http://engageny.org/wp-content/uploads/2012/05/Shifts-for-Students-and-Parents.pdf> Information for parents and students about the Shifts associated with the CCSS.
* <http://havefunteaching.com/> Various resources, including tools such as sets of Common Core Standards posters.
* <http://michellef.essdack.org/links> Numerous mathematics links.
* <http://illustrativemathematics.org/> Tasks that align with the MD CCSS.
* <http://www.aimsedu.org/Puzzle/categories/topological.html> Puzzles to challenge students of various ages.
* <http://www.insidemathematics.org/index.php/home> Mathematics resources for teachers.
* <http://www.graniteschools.org/depart/teachinglearning/curriculuminstruction/math/Pages/MathematicsVocabulary.aspx> Common Core vocabulary and vocabulary cards.

**Math Related Literature:**

* Ernst, Lisa Campbell, Sam Johnson and the Blue Ribbon Quilt

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