Mathematics

Geometry

Maryland College and Career Ready Standards

2018

MARYLAND STATE DEPARTMENT OF EDUCATION
EQUITY AND EXCELLENCE

Contents

★ Modeling Standard

(+) indicate additional mathematics that students should learn in order to take advanced courses

Introduction

The College and Career Ready State Standards for Mathematics (CCSSM) at the high school level specify the mathematics that all students should study in order to be college and career ready. The high school standards are listed in conceptual categories (number and quantity, algebra, functions, geometry, modeling, and probability and statistics). Consideration of how to organize the CCSSM high school standards into courses that provides a strong foundation for post secondary success was needed. To answer this charge, a group of experts, including state mathematics experts, teachers, mathematics faculty from two and four year institutions, mathematics teacher educators, and workforce representatives, were convened to develop Model Course Pathways in high school based on College and Career Ready State Standards for Mathematics (CCSSM). The model pathways can be found in Appendix A of the College and Career Ready State Standards for Mathematics.

After a review of these pathways, the superintendants of Maryland's LEA's voted to adopt the pathway reflected in this framework document which is referred to as the "Traditional Pathway". The "Traditional Pathway" consists of two algebra courses and a geometry course, with some data, probability and statistics included in each algebra course.

★ Modeling Standard
(+) indicate additional mathematics that students should learn in order to take advanced courses

How to Read the Maryland College and Career Ready Curriculum Framework for Geometry

This framework document provides an overview of the standards that are grouped together to form the units of study for Geometry. The standards within each unit are grouped by conceptual category and are in the same order as they appear in the College and Career Ready State Standards for Mathematics. This document is not intended to convey the exact order in which the standards within a unit will be taught nor the length of time to devote to the study of the unit.

The framework contains the following:

- **Units** are intended to convey coherent groupings of content.
- **Clusters** are groups of related standards**.** A description of each cluster appears in the left column.
- **Cluster Notes** are instructional statements which relate to an entire cluster of standards. These notes are placed in the center column above all of the standards in the cluster.
- **Essential Skills and Knowledge** statements provide language to help teachers develop common understandings and valuable insights into what a student must understand and be able to do to demonstrate proficiency with each standard. Maryland mathematics educators thoroughly reviewed the standards and, as needed, provided statements to help teachers comprehend the full intent of each standard. The wording of some standards is so clear, however, that only partial support or no additional support seems necessary.
- **Standards** define what students should understand and be able to do. **Notes** are instructional notes that pertain to just one standard. They are placed in the center column immediately under the standard to which they apply. The notes provide constraints, extensions and connections that are important to the development of the standard.
- **Standards for Mathematical Practice** are listed in the right column.
- ★ Denotes that the standard is a Modeling standard. Modeling standards are woven throughout each conceptual category.
- (+) indicates additional mathematics that students should learn to prepare for advanced courses.

Formatting Notes

- ***Red Bold****- information added by Maryland Educators.*
- ***Blue bold*** *– words/phrases that are linked to clarifications*
- ***Black bold underline****- words within repeated standards that indicate the portion of the statement that is emphasized at this point in the curriculum or words that draw attention to an area of focus*
- ***Black bold****- Cluster Notes-notes that pertain to all of the standards within the cluster*
- ***Green bold*** *– standard codes from other courses that are referenced and are hot linked to a full description*

★ Modeling Standard
(+) indicate additional mathematics that students should learn in order to take advanced courses

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5+ 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $\frac{y-2}{x-1} = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Modeling Standards

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

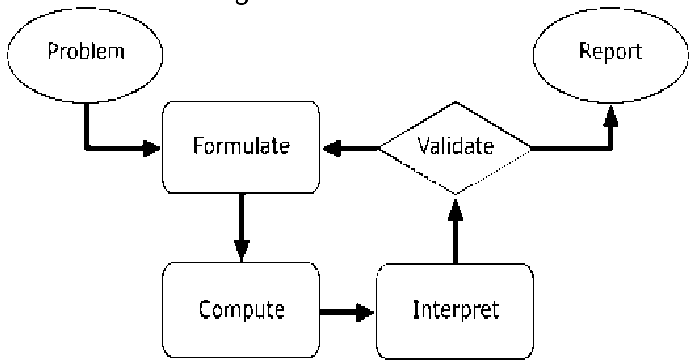
Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram.



It involves:

1. identifying variables in the situation and selecting those that represent essential features.
2. formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables.
3. analyzing and performing operations on these relationships to draw conclusions.
4. interpreting the results of the mathematics in terms of the original situation.
5. validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable.
6. reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol ★ .*

(+) Standards *The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+). All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.*

★ Modeling Standard

(+) indicate additional mathematics that students should learn in order to take advanced courses

Codes for College and Career Ready State Standards (Math) Standards – K – 12

Grades K-8

Domain Code	Domain Name	Applicable Grades
CC	Counting & Cardinality	K
EE	Expressions & Equations	6, 7, 8
F	Functions	8
G	Geometry	K, 1, 2, 3, 4, 5, 6, 7, 8
MD	Measurement & Data	K, 1, 2, 3, 4, 5
NBT	Number & Operations (Base Ten)	K, 1, 2, 3, 4, 5
NF	Number & Operations (Fractions)	3, 4, 5
NS	Number System	6, 7, 8
OA	Operations & Algebraic Thinking	K, 1, 2, 3, 4, 5
RP	Ratios & Proportional Relationship	6, 7
SP	Statistics & Probability	6, 7, 8
Modeling		
No Code		Not Determined

High School

Algebra (A)

Domain Code	Domain Name	Applicable Grades
A-APR	Arithmetic with Polynomial & Rational Expressions	8–12
A-CED	Creating Equations	8–12
A-REI	Reasoning with Equations & Inequalities	8–12
A-SSE	Seeing Structure in Expressions	8–12

Functions (F)

Domain Code	Domain Name	Applicable Grades
F-BF	Building Functions	8–12
F-IF	Interpreting Functions	8–12
F-LE	Linear, Quadratic & Exponential Models	8–12
F-TF	Trigonometric Functions	Not determined

Geometry (G)

Domain Code	Domain Name	Applicable Grades
G-C	Circles	Not determined
G-CO	Congruence	Not determined
G-GMD	Geometric Measurement & Dimension	Not determined
G-MG	Modeling with Geometry	Not determined
G-GPE	Expressing Geometric Properties with Equations	Not determined
G-SRT	Similarity, Right Triangles & Trigonometry	Not determined

Number & Quantity (N)

Domain Code	Domain Name	Applicable Grades
N-CN	Complex Number System	Not determined
N-Q	Quantities	Not determined
N-RN	Real Number System	8–12
N-VM	Vector & Matrix Quantities	Not determined

Statistics (S)

Domain Code	Domain Name	Applicable Grades
S-ID	Interpreting Categorical & Quantitative Data	8–12
S-IC	Making Inferences & Justifying Conclusions	Not determined
S-CP	Conditional Probability & Rules of Probability	Not determined
S-MD	Using Probability to Make Decisions	Not determined

Modeling

Domain Code	Domain Name	Applicable Grades
No Code		Not Determined

Traditional Pathway for Geometry

Traditional Pathway

The fundamental purpose of the course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school CCSS. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas, organized into five units are as follows.

Critical Area 1: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Critical Area 2: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem.

Critical Area 3: Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

Critical Area 4: Building on their work with the Pythagorean Theorem in 8th grade to find distances, students use a Cartesian coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, which relates back to work done in Algebra I.

Critical Area 5: In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane.

Overview of the Units of Study

Units	Clusters*	Standards
Unit 1 Congruence, Proof and Constructions	• Experiment with transformations in the plane • Understand congruence in terms of rigid motions • Prove geometric theorems • Make geometric constructions	• G.CO.1,2,3,4,5, 6,7,8,9,10,11,1 2 & 13
Unit 2 Similarity, Proof and Trigonometry	• Understand similarity in terms of similarity transformations • Prove theorems involving similarity • Define trigonometric ratios and solve problems involving right triangles • Apply geometric concepts in modeling situations • Apply trigonometry to general triangles	• G.SRT.1,2,3,4,5 ,6,7, 8,9+,10+ & 11+ • G.MG.1,2, &3
Unit 3 Extending to Three Dimensions	• Explain volume formulas and use them to solve problems • Visualize the relation between two-dimensional and three-dimensional objects • Apply geometric concepts in modeling situations	• G.GMD.1,3,&4 • G.MG.1
Unit 4 Connecting Algebra and Geometry through Coordinates	• Use coordinates to prove simple geometric theorems algebraically	• G.GPE. 4,5,6,&7
Unit 5 Circles With and Without Coordinates	• Understand and apply theorems about circles • Find arc lengths and areas of sectors of circles • Translate between the geometric description and the equation for a conic section • Use coordinates to prove simple geometric theorem algebraically • Apply geometric concepts in modeling situations	• G.C.1,2,3,4+&5 • G.GPE.1&4 • G.MG.1

***In some cases clusters appear in more than one unit within a course or in more than one course. Instructional Notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.**

Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

★ Modeling Standard

(+) indicate additional mathematics that students should learn in order to take advanced courses

Unit 1: Congruence, Proof, and Constructions

In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Cluster	Standard
Experiment with transformations in the plane.	**Cluster Note*: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.** G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. **Essential Skills and Knowledge** • **Ability to use mathematical vocabulary accurately** G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus **horizontal stretch**) **Essential Skills and Knowledge** • **Ability to see parallels between function transformations (F.BF.3) and geometric transformations** • **Knowledge that rigid transformations preserve the size and shape of a figure** G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. **Essential Skills and Knowledge** • **Ability to use appropriate vocabulary to describe rotations and reflections** • **Ability to use the characteristics of a figure to determine and then describe what happens to the figure as it is rotated (such as axis of symmetry, congruent angles or sides….)**

Cluster	Standard
Experiment with transformations in the plane. (Continued)	G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **Essential Skills and Knowledge** • **Ability to construct a definition for each term based upon a synthesis of experiences** G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. **Essential Skills and Knowledge** • **Ability to interpret and perform a given sequence of transformations and draw the result** **Ability to accurately use geometric vocabulary to describe the sequence of transformations that will carry a given figure onto another**

Cluster	Standard
Understand congruence in terms of rigid motions.	**Cluster Note*:* Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.** G.CO.6 Use geometric descriptions of **rigid motions** to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of **rigid motions** to decide if they are congruent. **Essential Skills and Knowledge** • **Ability to recognize the effects of rigid motion on orientation and location of a figure** • **Ability to use rigid motions to map one figure onto another** • **Ability to use the definition of congruence as a test to see if two figures are congruent** G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. **Essential Skills and Knowledge** • **Knowledge of vocabulary corresponding parts and the connection to the given triangles** • **Ability to identify the corresponding parts of two triangles** G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. **Essential Skills and Knowledge** • **Ability to recognize why particular combinations of corresponding parts establish congruence and why others do not**

Cluster	Standard
Prove geometric theorems.	**Cluster Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.** G.CO.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* **Note: This is an overarching standard that will be revisited throughout the course.** **Essential Skills and Knowledge** • **Ability to construct a proof using one of a variety of methods** G.CO.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* **Note**: Implementation of this standard may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for **G.C.3** in Unit 5. **Essential Skills and Knowledge** • **Ability to construct a proof using one of a variety of methods** G.CO.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.* **Essential Skills and Knowledge** • **Ability to construct a proof using one of a variety of methods**

★ Modeling Standard
(+) indicate additional mathematics that students should learn in order to take advanced courses

Cluster	Standard
Make geometric constructions.	**Cluster Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.** G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* **Essential Skills and Knowledge** • **Ability to use understanding of geometric concepts to establish a rationale for the steps/procedures used in completing a construction** G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. **Essential Skills and Knowledge** • **Ability to use understanding of geometric concepts to establish a rationale for the steps/procedures used in completing a construction**

Unit 2: Similarity, Proof, and Trigonometry

Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem.

Cluster	Standard
Understand similarity in terms of similarity transformations.	G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. **Essential Skills and Knowledge** • **Ability to connect experiences with dilations and orientation to experiences with lines** b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **Essential Skills and Knowledge** • **Ability to develop a hypothesis based on observations** G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using **similarity transformations** the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **Essential Skills and Knowledge** • **Ability to make connections between the definition of similarity and the attributes of two given figures** • **Ability to set up and use appropriate ratios and proportions** G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **Essential Skills and Knowledge** • **Ability to recognize why particular combinations of corresponding parts establish similarity and why others do not**

★ Modeling Standard
(+) indicate additional mathematics that students should learn in order to take advanced courses

Cluster	Standard
Prove theorems involving similarity.	G.SRT.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two sides proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* **Essential Skills and Knowledge** • **Ability to construct a proof using one of a variety of methods** G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **Note: This is an overarching standard that will be revisited throughout the course.** **Essential Skills and Knowledge** • **Ability to use information given in verbal or pictorial form about geometric figures to set up a proportion that accurately models the situation**

Cluster	Standard
Define trigonometric ratios and solve problems involving right triangles.	G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **Essential Skills and Knowledge** • **Ability to generalize that side ratios from similar triangles are equal and that these relationships lead to the definition of the six trigonometric ratios** G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles. **Essential Skills and Knowledge** • **See the skills and knowledge that are stated in the Standard.** G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★ **Essential Skills and Knowledge** • **See the skills and knowledge that are stated in the Standard.**

★ Modeling Standard
(+) indicate additional mathematics that students should learn in order to take advanced courses

<table>
<tr><th>Cluster</th><th>Standard</th></tr>
<tr><td>Apply geometric concepts in modeling situations.</td><td>Cluster Note: Focus on situations well modeled by trigonometric ratios for acute angles

G.MG.1 Use geometric shapes, their measures, and their properties to describe objects. (e.g., modeling a tree trunk or a human torso as a cylinder). ★

Note: This is an overarching standard that has applications in multiple units

Essential Skills and Knowledge
• See the skills and knowledge that are stated in the Standard.

G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ★

Essential Skills and Knowledge
• See the skills and knowledge that are stated in the Standard.

G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

Essential Skills and Knowledge
• See the skills and knowledge that are stated in the Standard.</td></tr>
</table>

★ Modeling Standard
(+) indicate additional mathematics that students should learn in order to take advanced courses

Cluster	Standard
Apply trigonometry to general triangles.	**Cluster Note: With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.** G.SRT.9 (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **Essential Skills and Knowledge** • **Ability to make connections between the formula $A = \frac{1}{2}(base)(height)$ and right triangle trigonometry** G.SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems. **Essential Skills and Knowledge** • **Ability to recognize when it is appropriate to use the Law of Sines and the Law of Cosines** G.SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). **Essential Skills and Knowledge** • **See the skills and knowledge that are stated in the Standard.**

★ Modeling Standard
(+) indicate additional mathematics that students should learn in order to take advanced courses

Unit 3: Extending to Three Dimensions

Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

Cluster	Standard
Explain volume formulas and use them to solve problems.	**Cluster Note: Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.** G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use **dissection arguments**, **Cavalieri's principle**, and **informal limit arguments**.* **Essential Skills and Knowledge** • **See the skills and knowledge that are stated in the Standard.** G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★ **Note: This is an overarching standard that has applications in multiple units** **Essential Skills and Knowledge** • **See the skills and knowledge that are stated in the Standard**

Cluster	Standard
Visualize the relation between two dimensional and three-dimensional objects.	G.GMD.4 Identify the shapes of two-dimensional **cross-sections of three dimensional objects**, and identify three-dimensional objects generated by rotations of two-dimensional objects. **Essential Skills and Knowledge** • **Ability to make connections between two-dimensional figures such as rectangles, squares, circles, and triangles and three-dimensional figures such as cylinders, spheres, pyramids and cones**

★ Modeling Standard
(+) indicate additional mathematics that students should learn in order to take advanced courses

Cluster	Standard
Apply geometric concepts in modeling situations.	G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★ **Note**: Focus on situations that require relating two- and three-dimensional objects, determining and using volume, and the trigonometry of general triangles. **Note: This is an overarching standard that has applications in multiple units.** Essential Skills and Knowledge • **Ability to connect experiences with this standard as it related to the two-dimensional shapes studied in Unit 2 to three-dimensional shapes**

★ Modeling Standard
(+) indicate additional mathematics that students should learn in order to take advanced courses

Unit 4: Connecting Algebra and Geometry Through Coordinates

Building on their work with the Pythagorean theorem in 8th grade to find distances, students use a Cartesian coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, which relates back to work done in Algebra I.

Cluster	Standard
Use coordinates to prove simple geometric theorems algebraically.	**Cluster Note: This unit has a close connection with the next unit. For example, a curriculum might merge G.GPE.1 and the Unit 5 treatment of G.GPE.4 with the standards in this unit. Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles.** G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle.* **Note: This is an overarching standard that has applications in multiple units** **Essential Skills and Knowledge** • **Ability to use distance, slope and midpoint formulas,…** G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). **Note**: Relate work on parallel lines in this standard to work on **A.REI.5** in High School Algebra I involving systems of equations having no solution or infinitely many solutions. **Essential Skills and Knowledge** • **See the skills and knowledge that are stated in the Standard.** G.GPE.6 Find the point on a **directed line segment** between two given points that partitions the segment in a given ratio. **Essential Skills and Knowledge** • **Ability to use the slope formula** G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★ **Note**: This standard provides practice with the distance formula and its connection with the Pythagorean theorem. **Essential Skills and Knowledge** • **See the skills and knowledge that are stated in the Standard.**

★ Modeling Standard

(+) indicate additional mathematics that students should learn in order to take advanced courses

Unit 5: Circles With and Without Coordinates

In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane.

Cluster	Standard
Understand and apply theorems about circles.	G.C.1 Prove that all circles are similar. **Essential Skills and Knowledge** • **See the skills and knowledge that are stated in the Standard.** G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* **Essential Skills and Knowledge** • **See the skills and knowledge that are stated in the Standard.** G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **Essential Skills and Knowledge** • **Ability to use concurrence of perpendicular bisectors and angle bisectors for the basis of the construction** G.C.4 (+) Construct a tangent line from a point outside a given circle to the circle. **Essential Skills and Knowledge** • **See the skills and knowledge that are stated in the Standard.**

★ Modeling Standard
(+) indicate additional mathematics that students should learn in order to take advanced courses

Cluster	Standard
Find arc lengths and areas of sectors of circles.	G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. **Note**: Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular Trigonometry in this course. **Essential Skills and Knowledge** • **See the skills and knowledge that are stated in the Standard.**

Cluster	Standard
Translate between the geometric description and the equation for a conic section.	G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean theorem; **complete the square** to find the center and radius of a circle given by an equation. **Essential Skills and Knowledge** • **See the skills and knowledge that are stated in the Standard.**

Cluster	Standard
Use coordinates to prove simple geometric theorems algebraically.	G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.; *for example prove or disprove that the point* *(1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).* **Note**: Include simple proofs involving **circles**. **Note: This is an overarching standard that has applications in multiple units** **Essential Skills and Knowledge** • **Ability to connect experience with coordinate proofs from Unit 4 to circles**

Cluster	Standard
Apply geometric concepts in modeling situations.	G.MG.1 Use geometric shapes, their measures, and their properties to describe objects. (e.g., modeling a tree trunk or a human torso as a cylinder). ★ **Note**: Focus on situations in which the analysis of **circles** is required. **Note: This is an overarching standard that has applications in multiple units** **Essential Skills and Knowledge** • **Ability to connect experiences from Unit 2 and Unit 3 with two-dimensional and three-dimensional shapes to circles**

★ Modeling Standard
(+) indicate additional mathematics that students should learn in order to take advanced courses