Dilations using Measurement

1. a. On a blank piece of paper place a point P in the upper right corner.

   b. Draw any triangle ABC below and to the left of the point P. Make sure the triangle is fairly small. Determine the lengths of the sides and the measures of the three angles in ΔABC.

   c. Draw three rays $\overrightarrow{PA}$, $\overrightarrow{PB}$, and $\overrightarrow{PC}$.

   d. Measure $\overline{PA}$, $\overline{PB}$, and $\overline{PC}$.

   e. Mark a point D on $\overrightarrow{PA}$ so that 2PA = PD.

   f. Mark point E on $\overrightarrow{PB}$ so that 2PB = PE.

   g. Mark point F on $\overrightarrow{PC}$ so that 2PC = PF.

   h. Connect the points D, E and F to form triangle DEF. Determine the lengths of the sides and the measures of the three angles in ΔDEF.

   i. Compare triangles ABC and triangle DEF. Write a statement about the two triangles.

2. a. On a second blank piece of paper place a point Q in the upper left corner.

   b. Draw any triangle ABC below and to the right of the point Q. Make sure the triangle is fairly large and toward the bottom of the page. Determine the lengths of the sides and the measures of the three angles in ΔABC.

   c. Draw three rays $\overrightarrow{QA}$, $\overrightarrow{QB}$, and $\overrightarrow{QC}$.

   d. Measure $\overline{QA}$, $\overline{QB}$, and $\overline{QC}$.

   e. Mark a point X on $\overrightarrow{QA}$ so that $\frac{1}{2}QA = QX$.

   f. Mark point Y on $\overrightarrow{QB}$ so that $\frac{1}{2}QB = QY$.

   g. Mark point Z on $\overrightarrow{QC}$ so that $\frac{1}{2}QC = QZ$.

   h. Connect the points X, Y and Z to form triangle XYZ. Determine the lengths of the sides and the measures of the three angles in ΔXYZ.

   i. Compare triangles ABC and triangle XYZ. Write a statement about the two triangles.

3. In problems 1 and 2 you created a transformation called a **dilation**.
   a. Does a dilation preserve lengths of sides and measures of angles?
   b. Is a dilation an isometry? Use mathematics to justify your answer.

4. In problem 1 point P was called the center of dilation and triangle DEF was the image of triangle ABC under a dilation.
   a. Name the center of dilation in problem 2.
   b. Name the image triangle under the dilation in problem 2.

Dilations using Measurement (Continued)

5. a. Place a point T in the center of a third piece of paper.
   b. Draw a large quadrilateral ABCD so that point T is in the interior of ABCD.
   c. Draw rays $\overrightarrow{TA}$, $\overrightarrow{TB}$, $\overrightarrow{TC}$, and $\overrightarrow{TD}$.
   d. Measure the lengths of $\overline{TA}$, $\overline{TB}$, $\overline{TC}$, and $\overline{TD}$.
   e. Mark points K, L, M, and N on $\overrightarrow{TA}$, $\overrightarrow{TB}$, $\overrightarrow{TC}$, and $\overrightarrow{TD}$ such that each point is the midpoint of $\overline{TA}$, $\overline{TB}$, $\overline{TC}$, and $\overline{TD}$ respectively.
   f. Draw the new quadrilateral KLMN.
   g. Are the two quadrilaterals are similar? Use mathematics to justify your answer.
   h. A scale factor for a dilation is the factor by which a figure is enlarged or reduced. What is the scale factor for the dilation that starts with quadrilateral ABCD and ends with quadrilateral KLMN?
   i. What is the center of the dilation for this transformation?

Activity

Materials needed: piece of colored paper, string, tape, meter sticks, wall and chair.

1. Tape the piece of colored paper to the wall. Label the corners A, B, C and D.
2. Cut four pieces of string each 3 meters long and tape one piece to each corner of the piece of colored paper.
3. Gently pull all four pieces of string together and tie the group to the back of a chair placed in front of the wall and colored paper. Make sure each piece of string is tight.
4. Measure 1 meter from the chair along each string and mark the point. With a piece of string, connect the points marked to form a rectangle.
5. What are the lengths of each side of the pre-image rectangle (colored paper) and each side of the image rectangle (string rectangle)?

6. What is the scale factor for this dilation?

7. What is the center of dilation?

8. Now mark a point on each of the strings so that the scale factor is $\frac{1}{2}$ and make a third rectangle.
9. Compare the third rectangle and the colored piece of paper. Are they similar? Use mathematics to justify your answer.
10. What is the center of dilation?

11. Where would you mark the string to have a scale factor of 2?

Answers:

1. i. ΔABC and ΔDEF are similar. The corresponding angles are congruent and the ratio of the lengths of the corresponding sides is the same, 1:2.

2. i. ΔABC and ΔXYZ are similar. The corresponding angles are congruent and the ratio of the lengths of the corresponding sides is the same, 2:1.

3. a. Dilations only preserve angle measure.
   b. A dilation is not an isometry. An isometry preserves both angle measure and length of sides, and a dilation only preserves angle measure.

4. a. The center of dilation in problem 2 is the point Q.
   b. ΔXYZ is the image triangle.

5. g. The two quadrilaterals are similar. The corresponding angles are congruent and the ratio of the lengths of the corresponding sides is the same.
   h. The scale factor is $\frac{1}{2}$.
   i. Point T is the center of dilation.

Activity

5. Answers will vary. The ratio of the sides of the colored paper to the lengths of string should be 3:1.
6. The scale factor is $\frac{1}{3}$.
7. The center of dilation is the location of the knot of string tied to the chair.
9. The two rectangles are similar because they have congruent angle measures and the ratio of the lengths of the corresponding sides is the same.
10. The center of dilation is still the point where the strings come together.
11. The string would have to go through the other side of the wall and the points would be twice the distance from the chair to the colored paper on the wall.