Quadrilaterals in the Coordinate Plane

1. Given: A (4, 3), B (8, 3), C (9, 10), and D (5, 10) are vertices of a quadrilateral. Is quadrilateral ABCD a parallelogram? Use mathematics to justify your answer. (graphing is not acceptable).

2. Verify ABCD is a parallelogram by choosing one pair of opposite sides and showing them parallel and congruent. Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation

Alternate assignment

3. If A(2, 1), B(5, 4), C(8, 12), and D(5, 9) are the vertices of a quadrilateral, verify that quadrilateral ABCD is a parallelogram. Use mathematics to justify your answer. (Do the problem 4 different ways.)

    1. Both pairs of opposite sides parallel.
    2. Both pairs of opposite sides congruent.
    3. One pair of sides congruent and parallel.
    4. Diagonals bisect each other.

Method 1: *Let's show that this is a parallelogram using the definition of parallelogram. Ask: So what do we need to show? Both pairs of opposite sides are parallel. We might be able to see this better if we make a rough sketch so we know which sides we are talking about.* After making the sketch of ABCD (this may be a rough sketch with no orientation to the given coordinates), have students decide which pairs of sides need to be parallel. Then have them find the slopes. (You can save time by dividing up the labor.)

After computing the slopes, they should find that the slope of $\overline{\mathrm{AB}}$ and $\overline{\mathrm{DC}}$ is 0, and the slope of $\overline{\mathrm{BC}}$ and $\overline{\mathrm{AD}}$ is 7.

Justification of method one: *Now that we have the slopes computed we need to write a sentence or two explaining what we did and why we did it. I used the slope formula to find the slopes of the sides of the quadrilateral. The slope of* $\overline{\mathrm{AB}} = \overline{\mathrm{BC}}$ *= 0 and the slope of* $\overline{\mathrm{BC}} = \overline{\mathrm{AD}}$ *= 7. Both pairs of opposite sides have the same slope so I know that these sides are parallel. Therefore ABCD is a parallelogram by definition: a parallelogram is a quadrilateral with two pairs of parallel sides.*

Method 2: *What is another way to prove that ABCD is a parallelogram?* One of your students will respond: *Both pairs of opposite sides are congruent.* Then: *Which sides would we want to get congruent?* (Same sides that we used before.) *Which formula will we use this time?* (distance formula). *So, we need to show?* (AB   DC and BC=   D). Have the students apply the distance formula. To save time, assign different groups the different lengths. After computing the lengths, they should have found that AB = DC = 4 and BC = AD = square root of 50, or approx. 7.07. Have the groups write their justification. Let them volunteer to share their work. *Justification: by applying the distance formula I found two pairs of congruent opposite sides:*

$$AB = \sqrt{0^2 + 4^2} = 4$$

$$DC = \sqrt{0^2 + 4^2} = 4$$

$$AD = \sqrt{7^2 + 1^2} = \sqrt{50}$$

$$BC = \sqrt{7^2 + 1^2} = \sqrt{50}$$

ABCD is a parallelogram by the theorem: If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Answers to alternate assignment

1. By using the slope formula, the slope of $\overline{AB}$ and $\overline{CD}$ is 1: the slope of $\overline{AD}$ and $\overline{BC}$ is $\frac{8}{3}$. Because the slopes of these pairs of sides are the same we can say ABCD is a parallelogram by definition: if both pairs of opposite sides are parallel then the quadrilateral is a parallelogram.

2. By using the distance formula, the length of $\overline{AB}$ and $\overline{CD}$ is $\sqrt{18}$ and the length of $\overline{AD}$ and $\overline{BC}$ is $\sqrt{73}$. Since both pairs of opposite sides are congruent we know ABCD is a parallelogram by the theorem: if both pairs of opposite sides are congruent then the quadrilateral is a parallelogram.

3. By using the slope formula, the slope of $\overline{AB}$ and $\overline{CD}$ is 1. The length of these same sides is $\sqrt{18}$. ABCD is a parallelogram by the theorem: if one pair of opposite sides is both parallel and congruent then the quadrilateral is a parallelogram.

4. By using the midpoint formula, the midpoints of diagonals $\overline{AC}$ and $\overline{BD}$ are the same: (5,6.5). ABCD is a parallelogram by the theorem: if the diagonals bisect each other then the quadrilateral is a parallelogram.