From the Experimental to the Theoretical

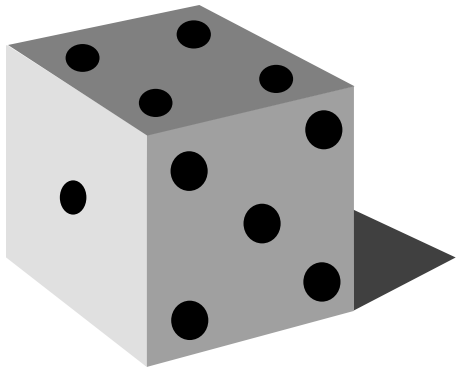
In this activity you will explore the relationship between theoretical and experimental probability. In the previous activities we explored **experimental probability**. We estimated probabilities using the results of data collected from samples of a larger population. **Theoretical probability** requires that we consider all possible outcomes. We determine the likelihood of an event by relating the number of favorable outcomes to the total number of outcomes.

Probability of event *E*,

$$P(E) = \frac{\text{number of outcomes making up event } E}{\text{total number of outcomes}}.$$

Suppose we collected many samples from the same population. The experimental probabilities determined from these samples should be close to the theoretical probabilities in the long run.

Consider the 6-sided number cube below. The six faces will appear at about the same rate when the cube is tossed, so the six possible outcomes should be equally likely.



1. Calculate the following theoretical probabilities.
   a. What is the theoretical probability that the cube will land on 6 when tossed?

   b. How often will the cube land on an even number?

   c. If you toss the cube one time, what is the likelihood that it will land on either 1 or 2?

   d. Suppose you toss the cube eight times and each time it lands on 3. What is the likelihood of getting a 3 on the next toss?

2. Calculate the following theoretical probabilities.

   a. P(landing on an odd number)

   b. $P(4)$

   c. $P(3,\ 4,\ 5,\ \text{or } 6)$

   d. $P(8)$

3. Obtain a number cube from your teacher and toss it 30 times. Record your results in the frequency table below.

Outcome	1	2	3	4	5	6	Total
Frequency							30

4. Your thirty tosses represent a sample from the population of all possible tosses. Estimate the following experimental probabilities using your sample results.

   a. likelihood of landing on an odd number

   b. likelihood of landing on 4

   c. likelihood of landing on 3, 4, 5, or 6

   d. likelihood of landing on 8

5. Compare the experimental probabilities in exercise 4 to the theoretical probabilities you determined in exercise 2.

6. Suppose you tossed the cube 10,000 times. How would your experimental estimates compare to the theoretical probabilities?

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Answer Key

1. a. $\frac{1}{6} = 0.167$

   b. The cube should land on an even number in about half of the tosses.

   c. $\frac{2}{6} = \frac{1}{3} = 0.333$

   d. $\frac{1}{6}$, the outcomes of the prior tosses do not affect the outcome of the ninth toss. We call this "independence."

2. a. $\frac{1}{2} = 0.50$

   b. $\frac{1}{6} = 0.167$

   c. $\frac{4}{6} = \frac{2}{3} = 0.667$

   d. 0

3. Answers will vary. The long-run frequencies (theoretical) are shown below.

Outcome	1	2	3	4	5	6	Total
Frequency	5	5	5	5	5	5	30

4. a. $\frac{15}{30} = \frac{1}{2} = 0.50$

   b. $\frac{5}{30} = \frac{1}{6} = 0.167$

   c. $\frac{20}{30} = \frac{2}{3} 0.667$

   d. 0

5. The experimental probabilities should be somewhat close to the theoretical probabilities, although some may be further off than others. Combining student/group results should yield experimental probabilities even closer to what we'd expect in the long run.

6. After 10,000 tosses the experimental probabilities should be extremely close to the theoretical probabilities.