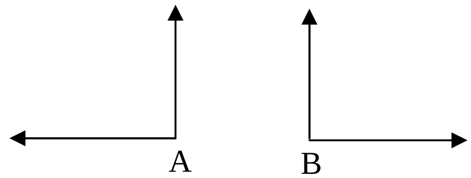
Paragraph Proofs

Use a paragraph proof to justify the following conjectures.

1. If two angles are both congruent and supplementary, then each angle is a right angle.

Given: $\angle A \cong \angle B$
$\angle A$ is supplementary to $\angle B$

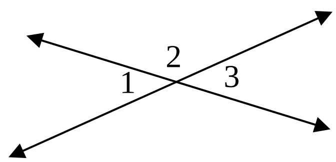
Prove: $\angle A$ is a right angle
$\angle B$ is a right angle



2. Vertical angles are congruent.

Given: $\angle 1$ and $\angle 3$ are vertical angles

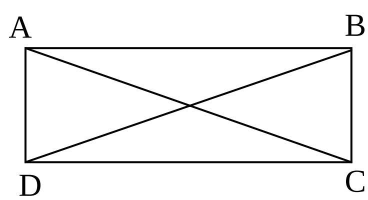
Prove: $\angle 1 \cong \angle 3$



3. The diagonals of a rectangle are congruent.

Given: Rectangle ABCD with diagonals $\overline{AC}$ and $\overline{BD}$

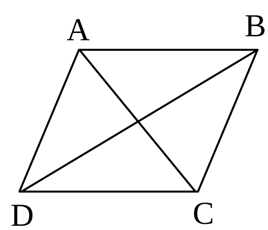
Prove: $\overline{AC} \cong \overline{BD}$



4. The diagonals of a rhombus bisect the angles.

Given: Rhombus ABCD with diagonals $\overline{AC}$ and $\overline{BD}$

Prove: $\overline{AC}$ bisects $\angle BAD$ and $\angle BCD$
$\overline{BD}$ bisects $\angle ADC$ and $\angle ABC$



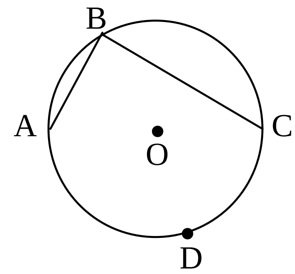
Paragraph Proofs (Continued)

5. Angles inscribed in a semicircle are right angles.

Given: $\angle B$ is inscribed in circle O

$\overset{\frown}{ABC}$ is a semicircle

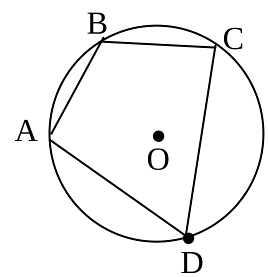
Prove: $\angle B$ is a right angle



6. If a quadrilateral is inscribed in a circle, then the opposite angles are supplementary.

Given: Quadrilateral ABCD is inscribed in circle O

Prove: $\angle A$ is supplementary to $\angle C$
$\angle B$ is supplementary to $\angle D$

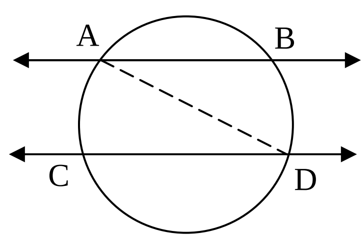


7. Parallel lines intercept congruent arcs on a circle.

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Prove: $\overset{\frown}{AC} \cong \overset{\frown}{BD}$

(Hint: Draw segment AD)



Answers:

1. $\angle A$ is congruent to $\angle B$ and $\angle A$ is supplementary to $\angle B$. Since the two angles are supplementary, their sum is 180º. Since they are congruent, they can be substituted for one another, meaning that $\angle A + \angle B$ is equal to 180º, but also that 2 times ($\angle B$) is equal to 180º. Then, $\angle B = 90º$ by the division property of equality. Since the two angles are congruent, $\angle A$ also = 90º. $\angle A$ and $\angle B$ are right angles by the definition of right angles.
2. $\angle 1$ and $\angle 3$ are vertical angles. Since they are vertical angles, there is an angle in between them, $\angle 2$, which is adjacent to both angles and supplementary to both angles. Since both $\angle 1$ and $\angle 3$ are supplementary to $\angle 2$, $\angle 1 + \angle 2 = 180º$ and $\angle 2 + \angle 3 = 180º$. $\angle 1 + \angle 2 = \angle 2 + \angle 3$ by the application of the transitive property of equality. $\angle 1$ and $\angle 3$ are congruent because to the subtraction property of equality.
3. ABCD is a rectangle with $\overline{AC}$ and $\overline{BD}$ as diagonals. Since ABCD is a rectangle, opposite sides $\overline{AB}$ and $\overline{CD}$ are congruent. In addition, $\overline{BC}$ and $\overline{DA}$ are congruent. Since ABCD is a rectangle, $\angle B$ and $\angle C$ are right angles, and both equal to 90º by the definition of right angles. Since both are equal to 90º, they are equal to one another by the transitive property. ΔABC and ΔBCD are congruent by the side-angle-side triangle congruence theorem. $\overline{AC}$ is congruent to $\overline{BD}$ by the definition of congruent triangles.
4. ABDC is a rhombus with diagonals $\overline{AC}$ and $\overline{BD}$. Since ABCD is a rhombus, all four sides are congruent. In addition, $\overline{AC} \cong \overline{AC}$ and $\overline{BD} \cong \overline{BD}$ by the reflexive property of congruence. ΔABC is congruent to ΔCDA and ΔBCD is congruent to ΔDAB by the side-side-side triangle congruence theorem. Therefore,

   $\angle BAC \cong \angle DAC$ $\qquad$ $\angle BCA \cong \angle DCA$

   $\angle ABD \cong \angle CBD$ $\qquad$ $\angle CDA \cong \angle ADC$

   by the definition of triangle congruence. $\overline{AC}$ bisects $\angle BAD$ and $\angle BCD$ and $\overline{BD}$ bisects $\angle ADC$ and $\angle ABC$ by the definition of angle bisectors.
5. $\angle B$ is inscribed in circle O and $\overset{\frown}{ABC}$ is a semicircle. The measure of arc ABC is 180º by the definition of a semicircle. The m$\angle B$ is 90º because the measure of an inscribed angle is half the measure of its intercepted arc. Therefore, by definition of a right angle, $\angle B$ is a right angle.

6. Quadrilateral ABCD is inscribed in circle O. There are 360º in a circle, so m $\overset{\frown}{ABC}$ + m $\overset{\frown}{CDA}$ = 360º and the m $\overset{\frown}{BCD}$ + m $\overset{\frown}{DAB}$ = 360º. By the division property of equality, $\frac{1}{2}$ m $\overset{\frown}{ABC}$ + $\frac{1}{2}$ m $\overset{\frown}{CDA}$ = 180º and

$$\frac{1}{2}\ m\overset{\frown}{BCD} + \frac{1}{2}\ m\overset{\frown}{DAB} = 180^\circ.$$

The $m\angle D = \frac{1}{2}\ m\overset{\frown}{ABC}$, $m\angle B = \frac{1}{2}\ m\overset{\frown}{CDA}$,

$m\angle A = \frac{1}{2}\ m\overset{\frown}{BCD}$, and $m\angle C = \frac{1}{2}\ m\overset{\frown}{DAB}$ because the

measure of an inscribed angle is one-half the measure of its intercepted arc. $m\angle D + m\angle B = 180^\circ$ and $m\angle A + m\angle C = 180^\circ$ by substitution. Therefore, $\angle A$ is supplementary to $\angle C$ and $\angle B$ is supplementary to $\angle D$ by definition of supplementary angles.

7. $\overline{AB}$ is parallel to $\overline{CD}$. Draw $\overline{AD}$. $\angle BAD \cong \angle CDA$ because if two parallel lines are cut by a transversal, the alternate interior angles are congruent. $\angle BAD = \angle CDA$ by the definition of congruent angles. $m\angle BAD = \frac{1}{2}\ m\overset{\frown}{BD}$ and $m\angle CDA = \frac{1}{2}\ m\overset{\frown}{AC}$ because the measure of an inscribed angle is one-half the measure of its intercepted arc. $\frac{1}{2}\ m\overset{\frown}{BD} = \frac{1}{2}\ m\overset{\frown}{AC}$ by substitution. The measure of $\overset{\frown}{BD}$ = m $\overset{\frown}{AC}$ by the multiplication property of equality. Therefore, $\overset{\frown}{AC} \cong \overset{\frown}{BD}$ by the definition of congruent arcs.