Coordinate Proofs

Use coordinate proofs to justify the following conditional statements.

1. The segment joining the midpoints of two opposite sides of a parallelogram is parallel to the other two sides.

2. The quadrilateral formed by joining, in order, the midpoints of the sides of a quadrilateral is a parallelogram.

3. The diagonals of a parallelogram bisect each other.

4. The diagonals of a rectangle are congruent.

5. The diagonals of a rhombus are perpendicular.

6. The mid-segment of a trapezoid is

   a. parallel to each of the bases of the trapezoid, and

   b. equal in length to one-half the sum of the lengths of the two bases.

Answers:

1. Students should set up a parallelogram (with coordinates (0, 0), (a, 0), (b, c), and (a + b, c) for example), find the midpoints of two opposite sides $\left(\left(\frac{a}{2},0\right)\text{and}\left(\frac{a+2b}{2},c\right)\text{in the example}\right)$ and then find slope of the two sides as well as the segment joining the midpoints to determine that they are all parallel.
2. Students should create a random quadrilateral, determine the midpoints of all four sides, find the slopes of the opposite sides, and find that opposite sides are parallel.
3. Students should create a parallelogram, find the midpoint of each diagonal, and determine that it is the same point.
4. Students should create a rectangle and find the distance between opposite vertices, showing that the lengths of the diagonals are the same.
5. Students should create a random rhombus (being sure that all four lengths are the same), then determine the slope between the opposite vertices. The product of the slopes should be –1 to show that they are perpendicular.
6. a. Students should create a trapezoid and find the midpoints of the non-parallel sides. Students should find the slope of the parallel sides as well as the slope of the segment connecting the midpoints to show that the three segments are parallel.
   b. Using the distance formula, students should show that the length of the midsegment is one-half the length of the sum of the two parallel bases.