Concurrent Lines in Triangles

Centroid

Step 1: A *median of a triangle* is _____.

Step 2: Draw a triangle on your paper as directed by your teacher.

Step 3: Construct the three medians of the triangle.

Step 4: *Concurrent lines* are three or more coplanar lines that intersect in the same point. Are the three medians of the triangle concurrent? If so, is the point of concurrency inside, outside, or on the triangle?

Step 5: The point of concurrency of the medians of a triangle is called the *centroid* of the triangle. The centroid divides each median into two segments. Use your patty paper or a compass to compare the lengths of these two segments on each of the medians. What is the relationship between the lengths of these two segments? What is the relationship between each of these segments and the median?

Step 6: Write a conjecture stating the relationship between the two segments of each median formed by the centroid of the triangle.

Step 7: Draw a large triangle on a piece of heavy paper provided by your teacher. Use scissors to cut out your triangle.

Step 8: Balance your triangle on the tip of your pencil. The point at which the triangle is balanced is called the *center of gravity* of the triangle. Label the center of gravity of your triangle as point G.

Step 9: Use a ruler to locate the midpoint of each side of the triangle. Draw the three medians of the triangle.

Step 10: What is the relationship between the center of gravity of your triangle and the intersection of the three medians of the triangle? Compare your results to the other members of your group.

Step 11: State a second conjecture about the point of concurrency of the medians of a triangle.

Circumcenter

Step 1: Draw a triangle on your paper as directed by your teacher.

Step 2: Construct the perpendicular bisector of each of the sides of the triangle.

Concurrent Lines in Triangles (Continued)

Circumcenter (Continued)

Step 3: Are the three perpendicular bisectors of the sides of the triangle concurrent? If so, is the point of concurrency inside, outside, or on the triangle?

Step 4: What is the relationship between the distances between the point of concurrency of the perpendicular bisectors of the sides of the triangle and each of the vertices of the triangle?

Step 5: Use your compass to construct a circle. Use the point of concurrency as the center of the circle. Set the radius of the compass so that it passes through one of the vertices of the triangle.

Step 6: Does your circle pass through the other two vertices of the triangle?

Step 7: A circle is circumscribed about a triangle if and only if it passes through each of the vertices of the triangle. Does the circle you drew in step 5 circumscribe your triangle? Use mathematics to justify your answer,

Step 8: The point of concurrency of the perpendicular bisectors of the sides of a triangle is called the circumcenter of the triangle. Why do think this name was selected as the point of concurrency?

Step 9: The circumcenter of a triangle is equidistant from the three _____ of a triangle.

Incenter

Step 1: Draw a triangle on your paper as directed by your teacher.

Step 2. Construct the three angle bisectors of the triangle.

Step 3: Are the three angle bisectors of the triangle concurrent? If so, is the point of concurrency inside, outside, or on the triangle?

Step 4: What is the relationship between the distances between the point of concurrency of the angle bisectors of the triangle and each of the sides of the triangle? (Remember: The distance between a point and a line segment is always the _____ distance.)

Concurrent Lines in Triangles (Continued)

Incenter (Continued)

Step 5: Use your compass to construct a circle. Use the point of concurrency as the center of the circle. Use the distance from the point of concurrency of the angle bisectors to a side of the triangle as the radius. Therefore, you must first construct a segment from the point of concurrency perpendicular to a side of the triangle.

Step 6: Is your circle tangent to each of the sides of the triangle? Use mathematics to justify your answer.

Step 7: A circle is inscribed within a polygon if and only if it touches each side of the polygon at exactly one point. Is the circle you drew in step 5 inscribed in your triangle?

Step 8: The point of concurrency of the angle bisectors of a triangle is called the *incenter* of the triangle. Why do think this name was selected as the point of concurrency?

Step 9: The incenter of a triangle is equidistant from the three _____ of a triangle.

Orthocenter

Step 1: Define *altitude* of a triangle.

Step 2: Draw a triangle on your paper as directed by your teacher.

Step 2. Construct the three altitudes of the triangle.

Step 3: Are the lines that contain the three altitudes of the triangle concurrent? If so, is the point of concurrency inside, outside, or on the triangle?

Step 4: The point of concurrency of the three altitudes of a triangle is called the orthocenter. Can you think of any other words that begin with the prefix *ortho*? What do you think the prefix *ortho* means?

Answers:

- Centroid
  - The median of a triangle is a segment that has as its endpoints a vertex of the triangle and the midpoint of the opposite side.
  - The medians of a triangle meet inside a triangle.
  - The lengths of the pieces of the median are in a 2:1 ratio – the point of concurrence is 2/3 of the distance from the vertex to the midpoint of the opposite side.
  - The concurrence point of the medians is the center of gravity of the triangle.
- Circumcenter
  - The point of concurrence of the perpendicular bisectors does not always meet inside the circle.
  - The point of concurrence of the perpendicular bisectors is equidistant from each vertex of the triangle.
  - Yes, the circle, using point of concurrence of the perpendicular bisectors as its center, circum-scribes the triangle.
  - It is called the circumcenter because it is the center of the circle that circumscribes the triangle, where each of the vertices of the triangle is on the circle.
  - The circumcenter of a triangle is equidistant from the three vertices of a triangle.
- Incenter
  - The point of concurrence of the angle bisectors falls inside of the triangle and is equidistant from the sides (perpendicular distance) of the triangle.
  - The circle that is drawn using the incenter is tangent to the three sides of the triangle because it touches each side at exactly one point. Therefore, it is inscribed in the triangle.
  - The point of concurrence of the angle bisectors is the center of the circle that is **in**side the triangle.
  - The incenter of a triangle is equidistant form the three sides of a triangle.
- Orthocenter
  - The point of concurrence of the altitudes of a triangle can be inside, outside, or on the triangle.
  - Other words with the ortho- prefix inculde orthopedic, orthodontic, orthodox, orthogonal, orthoscopic.
  - Ortho- means straight, regular, or upright.