Mathematics Toolkit: Grade 8 Objective 3.C.2.a

Standard 3.0 Knowledge of Measurement

Topic C. Applications in Measurement

Indicator 2. Analyze measurement relationships

Objective a. Use proportional reasoning to solve measurement problems

Assessment Limits:

Use proportions, scale drawings with scales as whole numbers, or rates using whole numbers or decimals (0 – 1000)

Table of Contents

Objective 3.C.2.a Tools

- Clarification
- Lesson Seeds
- Sample Item #1 - Selected Response (SR)

Clarification

Mathematics Grade 8 Objective 3.C.2.a Assessment Limit 1

A proportion is a statement of equality between two ratios. It is an equation whose members are ratios. A ratio is a comparison of any two quantities. The ratio of 2 to 4 can be stated as 2 out of 4, 2:4, or $\frac{2}{4}$. A ratio in which the comparison is between quantities with different units is a rate. Miles per hour, miles per gallon, square yards covered per gallons of paint are all examples of rates. When the rate is expressed as a comparison between x units and 1 unit, the rate is a unit rate. A speed limit of 55 mph is an example of a unit rate. The comparison is $\frac{55\text{miles}}{1\text{ hour}}$.

To develop the concept of proportion, begin by investigating simple problems built around ratios as *factors of change* or *unit rates*. The idea of a ratio as a rate is the underlying concept behind proportional reasoning. Proportional reasoning provides the underpinning for algebra and beyond. Change and rate of change is a key concept for all algebraic reasoning. Let's explore ratios in both contexts—as factors of change and as unit rates.

Classroom Example 1

Three apples cost 70¢. How much will a dozen cost?

Answer:

The ratio of number of apples to number of apples is 3 to 12 or 1 to 4. The *factor of change* in this problem is 4. To find the cost of 12 apples, multiply 70¢ by 4. The answer is 4 x 0.70 = $2.80. This problem could be done by finding the unit rate or cost per one apple, but $\frac{70}{3}$ does not simplify to a "friendly" number.

Classroom Example 2

Three dozen pencils cost $0.72. How much will 7 pencils cost?

Answer:

The *unit rate* or cost per one pencil is $\frac{72\text{cents}}{36\text{pencils}}$ or $\frac{2\text{cents}}{1\text{ pencils}}$.

7 pencils will cost 7 pencils x $\frac{2\text{cents}}{1\text{ pencils}}$ = 14 cents.

In both examples it is important that the method used is determined by the compatibility of the numbers involved. Also note that when we are comparing quantities using ratios as factors or unit rates, we are using a *multiplicative* comparison rather than an *additive* comparison. Most students up to this point have thought of comparisons as additive. For example:

In October there were 300 girls and 250 boys on the honor roll. In November there were 350 girls and 300 boys on the honor roll. Did the number of girls or the number of boys grow more?

If you are making an *additive comparison*, you could say they both grew by the same amount—50 students.
Based on a *multiplicative comparison*, the rate of growth for girls on the honor roll was $\frac{\text{increase}}{\text{original amount}} = \frac{50}{300} = \frac{1}{6}$ and the rate of growth for the boys on the honor roll was $\frac{\text{increase}}{\text{original amount}} = \frac{50}{250} = \frac{1}{5}$. Based on the greater rate of growth, the number of boys on the number roll increased proportionally more because $\frac{1}{5} > \frac{1}{6}$. Both answers are correct. The second answer, however, shows a proportional increase.

Students may have more experience in their mathematical background using additive comparisons and will need to investigate many examples to feel comfortable with multiplicative reasoning. Often times we use proportional reasoning and not realize it. More examples of proportional reasoning include finding the missing side with similar triangles, converting measurements, and using scales to enlarge or reduce a shape.

An important property of proportions is that $\frac{a}{b} = \frac{c}{d}$, if and only if $a \bullet d$ and $b \bullet c$. Is $\frac{14}{21} = \frac{2}{3}$? Is 14 • 3 = 21 • 2? Yes, 42 = 42. We can use this property when determining a missing number in a proportion. This property is commonly called the Cross-Product Property.

Classroom Example 3

If the ratio of girls to boys in a math club is 2 to 3 and there are 21 boys, how many girls would you expect to be in the club? We will solve this problem using the Cross-Product Property.

Answer:

Set up a proportion of equal ratios:

$$\frac{2}{3} = \frac{x}{21}$$
$$2 \bullet 21 = 3x$$
$$42 = 3x$$
$$14 = x$$

Classroom Example 4

A 2-ounce bag of M&M's contains 4 red M&M's. Using this ratio, how many red M&M's would you expect to be in a 16-ounce bag?

Method 1: Cross-Product Property

Answer:

$$\frac{\text{2 ounces}}{\text{4 red}} = \frac{\text{16 ounces}}{x}$$
$$2x = 4 \bullet 16$$
$$x = 32 \text{ M\&M's}$$

Method 2: Factor of Change

Answer:

The ratio of number of ounces to number of ounces is $\frac{16}{2}$ so that the factor of change is $\frac{16}{2}$ = 8. Multiply 4 red M&M's by the factor of change 8. The result is 32 M&M's.

Classroom Example 5

The scale on a map is 1 inch : 20 miles. The distance between your house and grandma's house is 5 inches on the map. How far, in miles, is your house from grandma's house?

Method 1: Cross-Product Property

Answer:

$$\frac{20 \text{ miles}}{1 \text{ inch}} = \frac{x}{5 \text{ inches}}$$
$$1 \cdot x = 5 \cdot 20$$
$$x = 100 \text{ miles to grandma's house}$$

Method 2: Factor of Change

Answer:

The ratio of miles to inches is $\frac{20}{1}$. To determine the number of miles that corresponds to 5 inches on the map, multiply 5 by the factor of $\frac{20}{1}$. 5 inches x $\frac{20 \text{ miles}}{1 \text{ inch}}$ = 100 miles.

Classroom Example 6

The Smith family is traveling cross country. They drive 396 miles in 9 hours. At this rate, how far will they drive in 24 hours?

Method 1: Cross-Product Property

Answer:

$$\frac{396 \text{ miles}}{9 \text{ hours}} = \frac{x}{24 \text{ hours}}$$
$$9 \cdot x = 396 \cdot 24$$
$$9x = 9504$$
$$x = 1056 \text{ miles}$$

Method 2: Unit Rate

Answer:

The rate the Smiths traveled per hour is $\frac{396 \text{miles}}{9 \text{ hours}} = \frac{44 \text{ miles}}{1 \text{ hour}}$. The number of miles traveled in 24 hours is $\frac{44 \text{miles}}{1 \cancel{\text{hour}}} \times 24 \cancel{\text{hours}}$ = 1056 miles. Notice how the units of measure are considered within the problem. It is important that students note the units of measure when setting up ratios and proportions. See the next example as an illustration.

Classroom Example 7

If a horse gallops 3 miles every 6 minutes, how many miles will he gallop in 30 seconds?

Answer:

Set up a proportion of equal ratios:

$$\frac{3\text{ miles}}{6\text{ minutes}} = \frac{x}{.5\text{ minutes}}$$

A correct proportion must involve the same units in each ratio. A common error would be to use 30 seconds as the denominator in the second ratio. Although 30 seconds is equivalent to .5 minutes, the first ratio has a denominator expressed in minutes, so the denominator of the second ratio should be expressed in minutes. When we solve this proportion, *x* is .25 miles. What would happen if we expressed the time in seconds in both ratios? Let's see:

Set up a proportion of equal ratios:

$$\frac{3\text{ miles}}{360\text{ seconds}} = \frac{x}{30\text{ seconds}}$$

$$360x = 90$$

$$x = .25\text{ miles}$$

Lesson Seeds

Mathematics Grade 8 Objective 3.C.2.a Assessment Limit 1

Activities

Show a scene from the movie "Zoolander". In one scene, one character, Mugato, makes a model of a school building to show to the main character, Zoolander. Zoolander gets upset because he believes that no one could possibly fit in the building because it is too small, not realizing that he is looking at a scale model. He thinks that the model is a full size representation of the school. This is an example of a humorous misunderstanding of proportional reasoning.

- Let the viewing of the scene lead into a discussion of proportional reasoning and why it is so important.
- Next, as a class, measure the dimensions of the classroom [An interesting extension would be to have some students measure in feet and inches and others in meters and then see which measurement system is easier to work in for scale drawings].
- Then ask students how they would fit the classroom on a standard sized sheet of paper and keep everything in proportional. Determine a correct scale and have students create a scale drawing of the classroom.
- Students can include other objects in the classroom as well to practice their proportional reasoning.

Ask students how the scale would change if they are asked to draw the whole school on that sheet of paper? Would the scale increase or decrease? How much? This type of questioning will led students into a rich discussion of proportional reasoning.

For additional practice, have students complete the same activity using other objects in the classroom.

Sample Item #1 - Selected Response (SR) Item

Mathematics Grade 8 Objective 3.C.2.a

Each week, Maurice deposits the same amount of money in his savings account. After 24 weeks, he saves $420. How much money does Maurice save per week?

A. $17.50
B. $21.00
C. $35.00
D. $41.50

Correct Answer:
A