Prisms: Areas and Volume

Objectives

Students will apply the formulas for surface area and volume of various prisms by first investigating the derivation of the formulas and then by applying the formulas in practice problems.

Core Learning Goals

2.3.2 Students will use techniques of measurement and will estimate, calculate and/or compare perimeter, circumference, area, volume, and/or surface area of three-dimensional figures and their parts.

Materials Needed

Worksheets, calculator, paper, tape, scissors, ruler, formula sheet, box of Kix or Mini Wheat cereal (or any type of cereal), an empty cereal box, three-dimensional models of a rectangular prism and a cylinder, and other prisms that you may have.

Pre-requisite Concepts Needed

Students will need to be able to determine the area of triangles, quadrilaterals, and polygons and use the special right triangle formulas to solve special right triangles. You may need to review finding the area of a regular hexagon by first finding the apothem of the hexagon.

Approximate Time

Two 45-minute lessons

Prisms: Areas and Volume

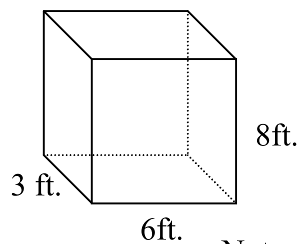
Lesson Plan

Warm-Up/Opening Activity

Show students three dimensional models of many prisms. Discuss what part of each model represents the surface area and the volume.

Using the model of the rectangular prism, show where the length, width, and height of the model are in order to determine the volume of the prism. As you do this, have the students note the formula for finding the volume of a rectangular prism on the HSA formula sheet. Next look at the surface area formula for a rectangular prism. Trace your finger along each dimension of the object as it comes in the formula. Ask the students to explain why the formula for surface are of a rectangular prism is written as it is.

Practice calculating the surface area and volume for the rectangular prism shown below:



Note: The figure is not drawn to scale.

$SA = 2lw + 2hw + 2hl = 2 \cdot 3 \cdot 6 + 2 \cdot 8 \cdot 6 + 2 \cdot 3 \cdot 8 = 180 \text{ ft}^2$
$V = l \cdot w \cdot h = 3 \cdot 6 \cdot 8 = 144 \text{ ft}^3$

Have students explain which unit to use for volume and which unit to use for surface area (cubic units, and square units)

Repeat the process using other models: a triangular prism, a hexagonal prism, and a cylinder. Be sure to ask the students 'why' the formula is written as it is. Show relationships between the formulas for different figures. For example, length 'times' width is the area of the base of a rectangle, therefore using the area of the base 'times' height is the same formula for volume in each of the other two prisms.

Warm-up

A net is a two-dimensional pattern of a three-dimensional figure. Nets can be folded to create cubes, rectangular prisms, cylinders, etc. Use the **Warm-up** worksheet to get students to look at nets, and have them pick the ones that would fold into a cube, a rectangular prism and a cylinder. If you have time, ask students to cut out the nets and actually do the folding.

Warm-up/Opening Activity (Continued)

Ask students to draw a net of one of the models that you have presented in class.

Answers:
1. Cube
2. None
3. Rectangular prism
4. None (be sure that students understand that it cannot be "twisted" to create a solid)
5. Cylinder (note that one of the sides is a little more than one unit wide, so a total of about 3.14)
6. None
7. Check student drawings

Development of Ideas

Initiate a discussion with students about the difference between surface area and volume (use the warm-up activity to help). Have a set of three dimensional figures to help guide your discussion. Show an example of a rectangular prism, a triangular prism and a cylinder (if you don't have good examples, just use a box and a can of soup). Remember, all figures we will discuss will be right prisms or right cylinders. They will only be regular when noted. Suggest that surface area will be the area that wrapping a gift would cover, and lateral area is the area that a label on a box or can may cover. Volume is the number of cubes that would completely fill the objects.

Activity One

Show students an empty cereal box. Ask them to estimate the volume, surface area and lateral area of the box.

Unfold the box, showing its net, and allow students to change their estimates if they want. (If you have enough boxes, give one to each group, otherwise, this can be a demonstration.)

Calculate the volume, surface area, and lateral area of the box. Get students to measure and to do the calculations. Suggestion: After measuring, let one-third of the class do each calculation (volume, surface area, and lateral area)- remember to use the HSA formula reference sheet.

Development of Ideas (Continued)

Worksheet: **Real World Applications**

Answers:

1. $V = l \cdot w \cdot h = 10 \cdot 17 \cdot 12 = 2{,}040 \text{ cm}^3$

2. Triangular prism: $V = \left(\frac{1}{2}\right)(12)(10.4)(17)$

   $= 1060.8 \text{ in}^3$

   Hexagonal prism: $V = Bh = \left(\frac{1}{2}\right) aP \cdot h$

   $= \left(\frac{1}{2}\right)(5.2)(36)(17)$

   $= 918\sqrt{3} \approx 1590 \text{ cm}^3$

   The volumes are not equal, the hexagonal prism is larger.

3. Triangular prism:

   $SA = \frac{1}{2}bh + \frac{1}{2}bh + 3lh$

   $= \left(\frac{1}{2}\right)(12)(10.4) + \left(\frac{1}{2}\right)(12)(10.4) + (3)(12)(17)$

   $= 736.8 \text{ in}^2$

   Hexagonal prism:

   $SA = \frac{1}{2}aP + \frac{1}{2}aP + 6lh$

   $= \left(\frac{1}{2}\right)(3\sqrt{3})(36) + \left(\frac{1}{2}\right)(3\sqrt{3})(36) + 6(6)(17)$

   $\approx 799 \text{ in}^2$

Activity Two – Investigation of volume and surface area of a rectangular prism

Worksheet: **Volume and Surface Area of a Rectangular Prism**

Each group of students needs an 8.5 in. x 11 in. piece of paper. Ask each group to create a box without a lid. Have one group cut a ½" square out of each corner, another a 1 in. square, and another with 1.5 in. squares. Continue until you get to 4in. if you have that many groups. Give each group tape so they can tape the sides together to make the box.

Development of Ideas (Continued)

Activity Two (Continued)

Initiate a discussion about why a group could not create a box using a 4.5 in. square.

Each group is to estimate then calculate the volume, and surface area of their 'box' (pretend the box has a top) completing the appropriate row in the table. Ask the students if the volume or surface areas of the boxes will be the same and justify their conclusion.

Ask students to show their boxes to each other. Have the students list the boxes in order from least to greatest by what they predict the volumes to be. Come to a consensus that the class can live with.

Fill the largest-predicted volume box with cereal. Poor that cereal into the next largest. Discuss what should happen if the volume is in fact larger (the cereal should flow over the smaller box). Continue on down the line and fix the order as necessary.

Have the students complete the table on the worksheet and mathematically verify their results.

Reflection: What dimension helped to determine the box with the greatest volume?

Answers:

Side of square	Length (in.)	Width (in.)	Height (in.)	Surface area (sq. in)	Volume (cu. in.)
0.5 in.	10 in	7.5 in.	0.5 in.	167.5	37.5
1.0 in	9 in.	6.5 in	1.0 in	148	58.5
1.5 in	8 in	5.5 in.	1.5 in	128.5	66
2.0 in.	7 in	4.5 in.	2.0 in.	109	63
2.5 in.	6 in	3.5 in.	2.5 in.	89.5	52.5
2.0 in.	5 in.	2.5 in.	3.0 in.	70	37.5
3.5 in.	4 in.	1.5 in.	3.5 in.	50.5	21
4.0 in	3 in	0.5 in	4.0 in	31	6

1. If you tried to cut a 4.5 in. square from each side, you would be cutting 2 x 4.5 = 9 in. from the paper. The paper is only 8.5 in. wide.
2. Answers will vary before the class sees the results of the other groups. After they have shared information they will see that the 8 x 5.5 x 1.5 box has the greatest volume.

Development of Ideas (Continued)

Answers to Activity Two (Continued)

3. Answers will vary before the class sees the results of the other groups. After they have shared information they will see that the 10 x 7.5 x 0.5 box has the greatest surface area. This makes sense since it has the least amount of paper cut from the corners.

Activity Three – Investigation of volume and lateral area of a cylinder
This activity can be done as a group, or as a demonstration

Use a sheet of 8½" by 11" paper to create a cylinder by joining the top and bottom edges. The edges need to meet exactly, with no gaps or overlaps.

Use a second sheet of paper the same size to make a different cylinder, this time joining the left and right edges together. Mark the tall one Cylinder A and the other B.

Ask students if they think the surface areas are the same for each cylinder (pretend each has a top and bottom). Use mathematics to justify your answer. (Use this time to reinforce that the lateral sides of a cylinder are a rectangle.)

Ask students if they think the volumes are the same for each cylinder. Use mathematics to justify your answer.

Place cylinder B on a flat surface (you might want to use a box lid) and place A inside it. Fill cylinder A with cereal. Allow students to adjust their predictions as necessary. Slowly lift cylinder A so that the cereal falls into cylinder B. Make the conclusions: Since the cereal does not fill cylinder B, then the volume of B is larger than A.

Reflection: Justify why the two cylinders did not hold the same volume.

Answer: Since V = $\pi \bullet r \bullet r \bullet h$, the radius has a bigger effect on the volume than the height, so the cylinder with the greatest radius will have the greatest volume.

Worksheet: **Volume of Paper Cylinders**

Show students a transparency of the cylinders on the worksheet, and ask them to predict the prism with the greatest volume. Have the students justify their answers by completing the table on the worksheet.

Development of Ideas (Continued)

Activity Three (Continued)

Answers:

A. $C = 11 + 11 = 22$ in
$C = 2\pi r = 22$ $\quad r \approx 3.5$ in
$V = \pi r^2 h = (\pi)(3.5)^2(4.25) \approx 163.6$ in³

B. $C = 11 = 2\pi r$ $\quad r \approx 1.75$ in
$V = \pi r^2 h = (\pi)(1.75)^2(8.5) \approx 81.8$ in³

C. $C = 8.5 = 2\pi r$ $\quad r \approx 1.35$ in
$V = \pi r^2 h = (\pi)(1.35)^2(11) \approx 63$ in³

D. $C = 8.5 \div 2 = 4.25$
$C = 4.25 = 2\pi r$ $\quad r \approx 0.676$ in
$V = \pi r^2 h = (\pi)(0.676)^2(22) \approx 31.6$ in³

A has the greatest volume because it has the largest radius.

Worksheet: **Determining Surface Area and Volume of Prisms Worksheets A** and **B**

These two sheets can be used in many different ways. You can assign both worksheets to all of the students or allow students to complete different worksheets and share their work with students that did not complete that worksheet.

Worksheet A Answers:

1. Rectangular prism: $V = l \cdot w \cdot h = 20 \cdot 20 \cdot 39$
$= 15{,}600$ ft³

Triangular prism: $V = Bh = \frac{1}{2} \cdot 20 \cdot 21 \cdot 20$
$= 4{,}200$ ft³

Total volume is $15{,}600 + 4{,}200 = 19{,}800$ ft³
$19{,}800 \div 144 = 137.5$

2. $SA = 2\left(\frac{1}{2}bh\right) + P \cdot w = 2\left(\frac{1}{2}(4.6)(2.77)\right) + 11.8 \cdot 7$
$= 95.34$

$$h = \sqrt{\left((3.6)^2 - (2.3)^2\right)} \approx 2.77$$

3. $V = 10 \cdot 8 \cdot 6 = 480$
$480 \cdot 1.5 = 720$

4a. $SA = 2B + Ch = 2(8^2 \cdot \pi) + (2 \cdot 8 \cdot \pi)h = 320\pi$
$320\pi - 128\pi = 16\pi h$
$12 = h$

Development of Ideas (Continued)

A Answers (Continued)

4b. $V = \pi \cdot 8^2 \cdot 12 = \pi \cdot r^2 \cdot 6$

$768\pi = 6\pi r^2$

$11.3 = r$ Therefore $d = 22.6$

5. $V = \pi r^2 h = \pi \cdot 2^2 \cdot 240 = 960\pi \approx 3016$

6. $SA(A) = 2lw + 2hw + 2lh$

$= 2(8)(2.5) + 2(8)(2.5) + 2(8)(8)$

$= 208 \text{ in}^2$

$SA(B) = 2lw + 2hw + 2lh$

$= 2(8)(2) + 2(10)(2) + 2(8)(10)$

$= 232 \text{ in}^2$

Box A has the least surface area and uses less cardboard.

Worksheet B Answers:

1. $V = \frac{1}{2} aP \cdot h = \frac{1}{2}(8\sqrt{3})(96)(53)$

$= 20{,}352\sqrt{3} \approx 35,251$

2. $V = s^3 = 1{,}728 \text{ yd}^3$

$s = 12$ Therefore, the area is 144

3. $V = Bh = 100 = \pi \cdot 4^2 \cdot h$ $h \approx 1.989 \approx 2$

4. $SA = B + LA = \frac{1}{2} aP + 6lw$

$= \frac{1}{2}(60\sqrt{3})(720) + 6(120)(80)$

$= 21{,}600\sqrt{3} + 57{,}600 \approx 95000$

5. $V = lwh = (15)(10)(3) = 450 \text{ in}^3$

$V = \pi r^2 h = \pi r^2 (15) = 450$

$r^2 = 9.55$

$r = 3.1$

6. When the radius of a cylinder is doubled, the volume is increased four times. A student can demonstrate this by using examples. The student begins with $V = \pi r^2 h$. If the radius is doubled, the new radius is $2r$. The new volume is $V = \pi (2r)^2 h = 4 \pi r^2 h$.

Prisms: Areas and Volume

Closure

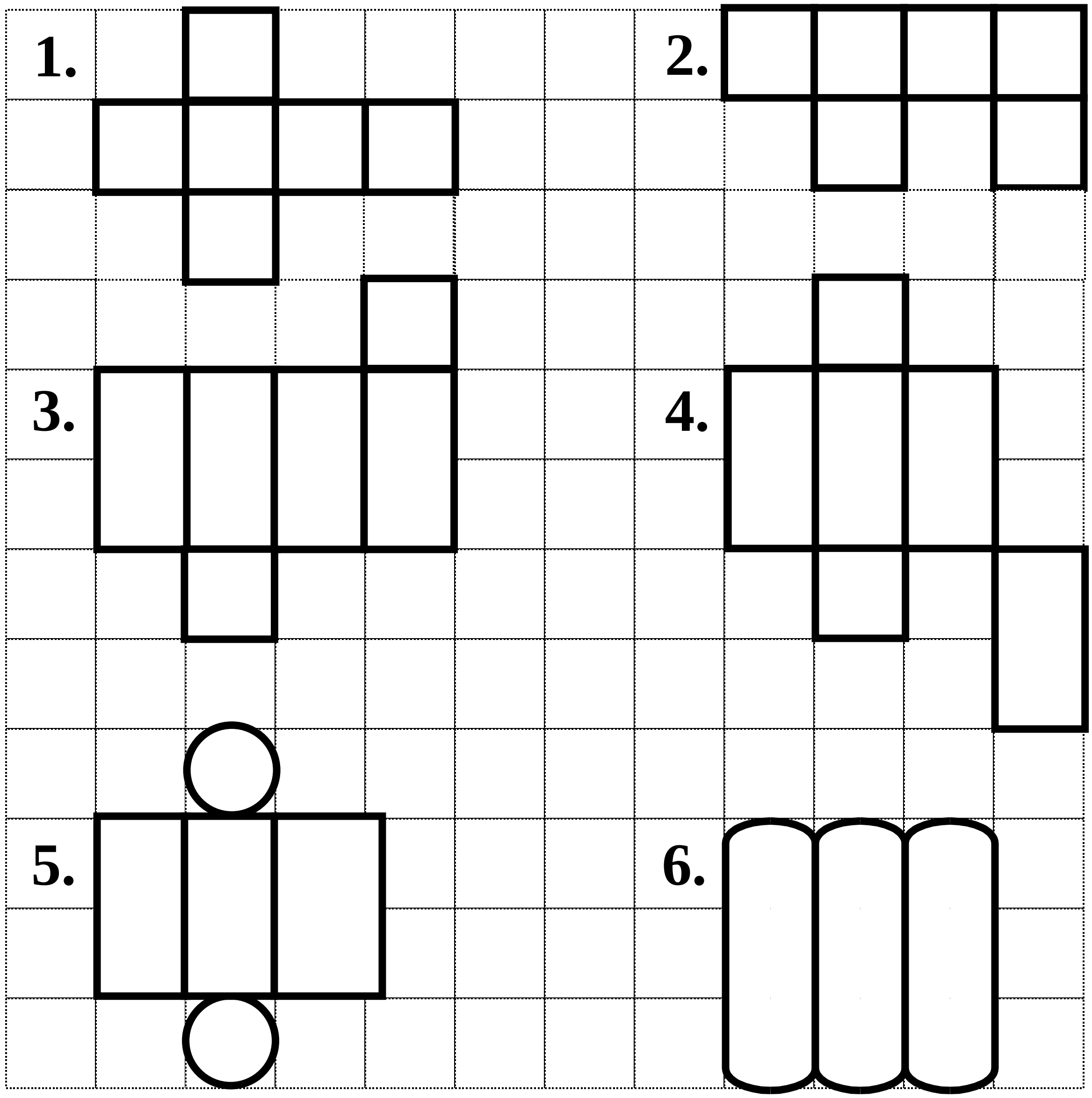
Restate the objective. Ask students to explain in writing how the objective was met in their own words.

Supplemental/Follow-Up Materials

Practice in class or at home using surface area and volume problems from the 2000, 2001, and 2002 HSA Public Release Versions.

Prisms: Areas and Volume

Warm-up



The nets above could be a net for a cube, a cylinder, or a rectangular prism. Write the correct name of the solid beside the net that could be folded into one of these solids. Some of these nets can not be folded into any of these solids. Explain what would need to change in order for these nets to become a net for a cube or cylinder or a rectangular prism.

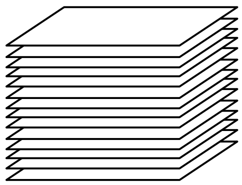
7. Think about some other nets for cubes, cylinders, or rectangular prisms. Draw them.

8. If you are not sure if one of these nets is for a cube, cylinder, or rectangular prism, cut it out and try to fold it into one of these solids.

Prisms: Areas and Volume

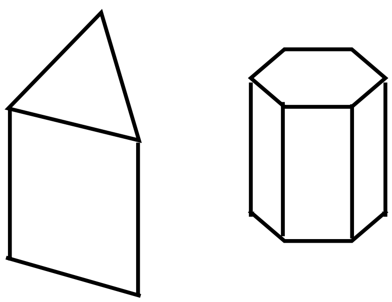
Real World Applications

1. A stack of 'Thank You' cards is piled 12 cm tall. These cards measure 10 cm by 17 cm. What is the volume of the box needed to store these cards? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.



Note: The figure is not drawn to scale.

2. One child's toy is in the shape of a triangular prism. The base is an equilateral triangle with sides of 12 inches. Another toy is a regular hexagonal prism. The side lengths of the base of the toy is 6 in. If the height of both toys is 17 inches, do they have equal volume? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.



Note: The figures are not drawn to scale.

3. What is the minimum number of square inches of wrapping paper needed to cover each of the above mentioned toys? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

Prisms: Areas and Volume

Volume and Surface Area of a Rectangular Prism

Directions: Each group has an 8.4 in. x 11 in. piece of paper. Create a box without a lid from the sheet of paper by cutting a square of the designated size from each corner of the paper and then folding the paper to make the box. Complete the row in the table below for the size square that you are assigned to cut out from the corner of the paper.

Side of square	Length (in.)	Width (in.)	Height (in.)	Surface area (sq. in)	Volume (cu. in.)
0.5 in.					
4.0 in.					
1.5 in.					
3.0 in.					
2.5 in.					
5.0 in.					
3.5 in.					
4.0 in					

1. Why can't you cut a 4.5 inch square from each corner of the paper?

2. Should all the boxes have the same volume? Use mathematics to justify your answer.

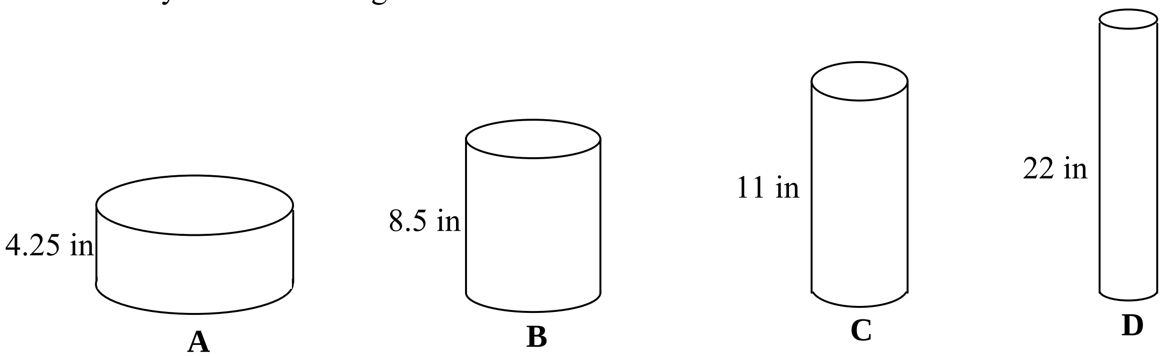
3. Should all the boxes (with the addition of the lid) have the same surface area? Use mathematics to justify your answer.

Prisms: Areas and Volume

Volume of Paper Cylinders

Each lateral surface was made using one entire sheet of $8\frac{1}{2}$" by 11" paper.

Choose the cylinder with the greatest volume.



Note: The figures are not drawn to scale.

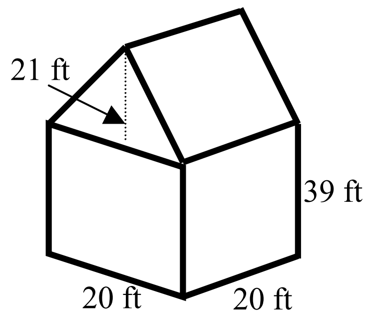
Directions: Complete the table for each cylinder.

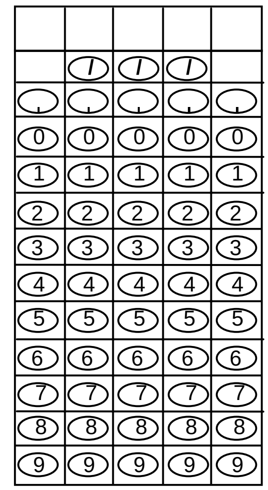
Cylinder	Circumference	Radius	Volume
A	11 + 11 = 22		
B			
C			
D			

Prisms: Areas and Volume

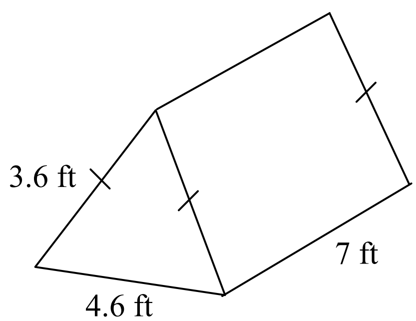
Determining Surface Area and Volume of Prisms: Worksheet A

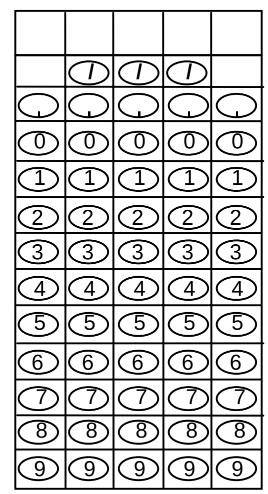
1. The heating system for the building shown below can raise the temperature 12° F at a rate of 144 cubic feet per minute. At this rate, how many minutes will it take to raise the temperature of the entire building 12°?



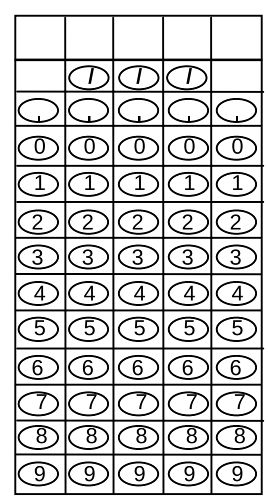


2. Suppose a tent in the shape of an isosceles triangular prism is resting on a flat surface. What is the surface area, in square feet, of the entire tent?



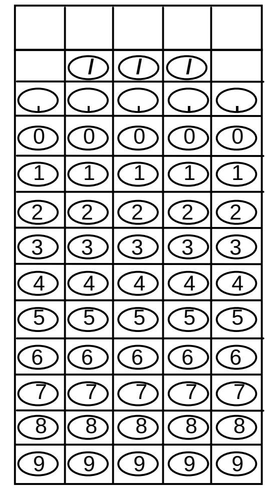


3. A cubic foot of ocean water contains about 1.5 pounds of salt. An aquarium is filled with ocean water. What is the weight, in pounds, of salt in the aquarium if the aquarium is a rectangular prism measuring 10 feet by 8 feet by 6 feet?

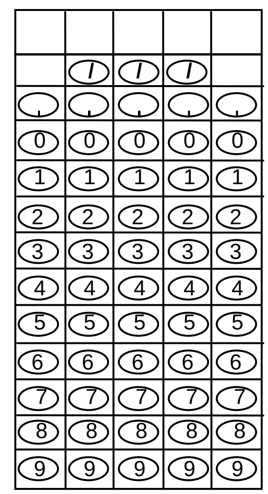


Determining Surface Area and Volume of Prisms: Worksheet A (Cont.)

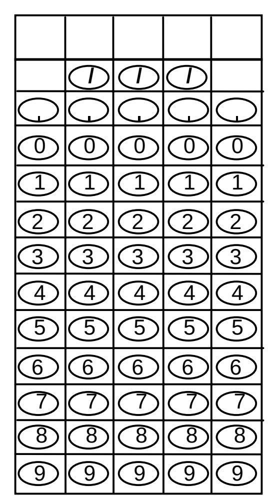
4a. Farmer Brown has a silo to store grain that is in the shape of a cylinder. The surface area of this silo is approximately 320π and the radius is 8 feet. What is the height, in feet, of his silo?



4b. Farmer Jones wants to build a silo to be the same volume as Farmer Brown's (see problem 4a), but half its height. What must be the diameter, in feet, of Farmer Jones' silo in order to meet these conditions?

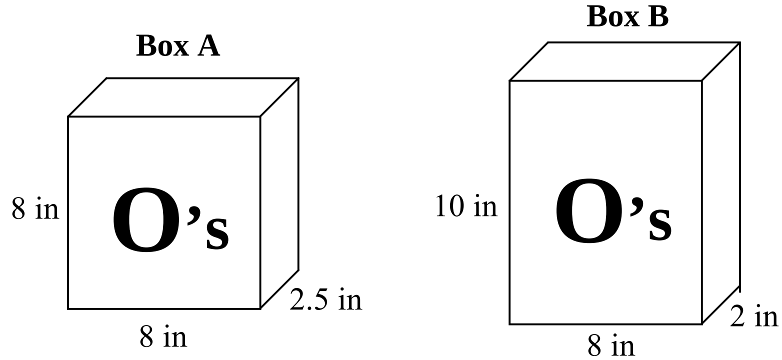


5. A straw is 24 cm long and 4 mm in diameter. How much liquid, in cubic mm, can the straw hold?



Determining Surface Area and Volume of Prisms: Worksheet A (Cont.)

6. BCR General Moles Cereal Company is choosing between two box designs with dimensions shown below. The company would like to keep the amount of cardboard used for the box to a minimum because it is better for the environment. The company must choose between two box designs.



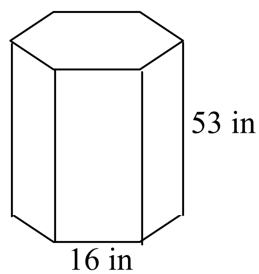
Note: The figures are not drawn to scale.

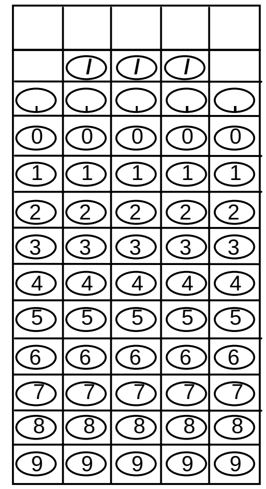
Which box uses less cardboard? Use mathematics to justify your answer.

Prisms: Areas and Volume

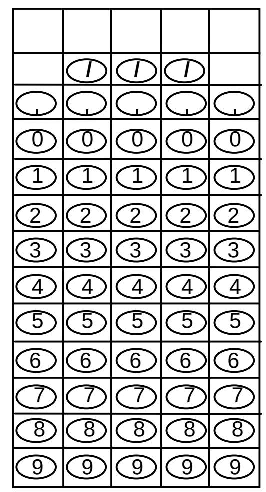
Determining Surface Area and Volume of Prisms: Worksheet B

1. An aquarium is in the shape of a hexagonal prism. Each side of its base is 16 inches. The aquarium is 53 inches tall. What is the volume, in cubic inches, of the aquarium?

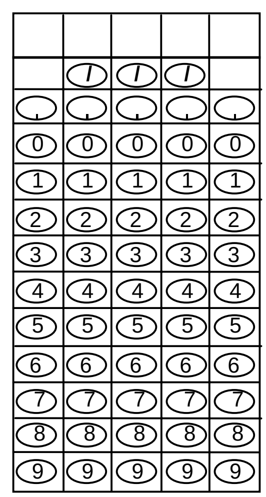




2. A pool in the shape of a cube has a volume of 1,728 yds^3. The pool's owner needs to paint the cover of his pool. If the cover of the pool fits exactly on the top of the pool, what is the area, in square yards, that the owner needs to paint?



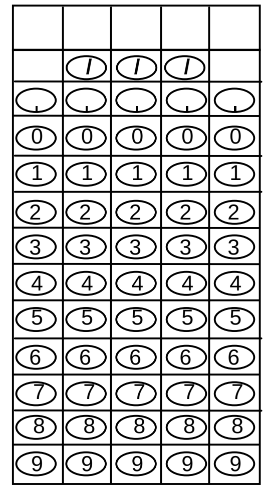
3. A cylindrical oil tank has a radius of 4 feet and holds 100 cubic feet of oil. What is the height, in feet, of the tank?



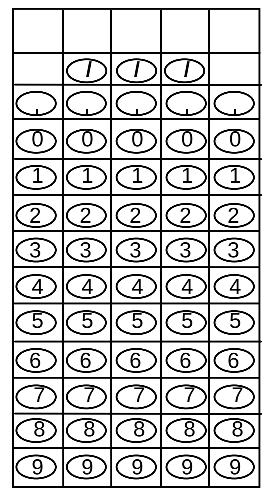
Prisms: Areas and Volume

Determining Surface Area and Volume of Prisms: Worksheet B (Cont.)

4. Dentyne Arena is in the shape of a regular hexagonal prism. The interior walls and ceiling are to be painted. The length of each side of the arena is 120 feet and the height of each wall is 80 feet. What is the area, in square feet, to be painted? Round your answer to the nearest 100 ft^2.



5. A cylindrical glass is full of water and is to be poured into a rectangular pan. The base of the pan is 15 inches by 10 inches and the height of the pan is 3 inches. The height of the cylinder is 15 inches. What is the radius of the cylinder, in inches, so that the water fills the pan but does not spill over?



6. ECR If the radius of a cylinder is doubled, what effect does the doubling have on the volume of the cylinder? Use mathematics to justify your answer.