Area and Volume of Similar Figures

Objective

The student will be able to determine and apply the relationship between perimeter, circumference, area, surface area, and/or volume of similar figures and their parts.

Core Learning Goal

2.3.2 The student will use techniques of measurement and will estimate, calculate, and/or compare perimeter, circumference, area, volume, and/or surface area of two- and three-dimensional figures and their parts.

Materials Needed

Worksheets, centimeter cubes, HSA formula reference sheet

Pre-requisite Concepts Needed

Students will need experience with calculating perimeter, circumference, area and volume, the Pythagorean Theorem, and identifying solid figures

Approximate Time

Two 45 minute lessons
Area and Volume of Similar Figures

Lesson Plan – Perimeter and Area of Similar Figures

Essential Question

What is the relationship between perimeter, circumference, area, surface area, and/or volume of similar figures and their parts?

Warm-Up/Opening Activity

Recall the relationship between scale factor and perimeter ratios of similar figures.

Student Guide: Perimeter and Area of Similar Figures (#1-4)

Answers:

1. a. \( \frac{9}{6} \) or \( \frac{3}{2} \)  
   b. \( \frac{9}{6} = \frac{3}{x} \)  
   c. \( P(ABCD) = 24 \) and \( P(MNOP) = 16 \)  
   d. \( \frac{24}{16} \) or \( \frac{3}{2} \)

2. a. \( \frac{16}{8} \) or \( \frac{2}{1} \)  
   b. \( \frac{16}{8} = \frac{6}{x} \)  
   c. \( 6^2 + 8^2 = (EG)^2 \)  
   \[ 3^2 + 4^2 = (HJ)^2 \]  
   \[ \Delta EFG: P = 10 + 10 + 16 = 36 \]  
   \[ \Delta HIJ: P = 5 + 5 + 8 = 18 \]  
   d. \( \frac{36}{18} \) or \( \frac{2}{1} \)

3. a. \( \frac{5}{2} \)
   b. \( \bigcirc A = 2(5) \pi = 10\pi \)  
   \( \bigcirc B = 2(2) \pi = 4\pi \)
   c. \( \frac{10\pi}{4\pi} \) or \( \frac{5}{2} \)

4. The ratio of the sides is the same as the ratio of the perimeters (circumferences).
Area and Volume of Similar Figures

Development of Ideas

Activity One

Investigate the relationship between scale factor and area ratios of similar figures.

Worksheet: Perimeter and Area of Similar Figures (#5-9)

Answers:

5. a. \( \frac{3}{2} \)
   
b. \( A \ (ABCD) = 9(3) = 27 \quad A \ (MNOP) = 6(2) = 12 \)
   
c. \( \frac{27}{12} \) or \( \frac{9}{4} \)

6. a. \( \frac{2}{1} \)
   
b. \( \triangle EFG : \frac{1}{2}(16)(6) = 48 \quad \triangle HIJ : \frac{1}{2}(8)(3) = 12 \)
   
c. \( \frac{48}{12} \) or \( \frac{4}{1} \)

7. a. \( \frac{5}{2} \)
   
b. \( \Theta A = \pi(5)^2 = 25\pi \quad \Theta B = \pi(2)^2 = 4\pi \)
   
c. \( \frac{25\pi}{4\pi} \) or \( \frac{25}{4} \)

8. The ratio of the areas of the similar figures equals the square of the ratio of the sides.

9. If two similar polygons or circles have lengths of corresponding sides (or radii) in the ratio of \( \frac{a}{b} \), then their areas are in the ratio of \( \frac{a^2}{b^2} \) or \( \left(\frac{a}{b}\right)^2 \).

Activity Two

Apply area ratios of similar figures.

Worksheet: Area Ratios of Similar Figures

Answers: 1. b. Answers will vary
   
c. The new area is four times the original area.
Area and Volume of Similar Figures

Development of Ideas (Continued)

Answers for Activity Two (Continued)

2. a. \( \frac{7}{3} \)  
   b. \( \frac{7}{3} \)  
   c. \( \frac{49}{9} \)

3. a. perimeter ratio: \( \frac{12}{15} \) or \( \frac{4}{5} \)  
   area ratio: \( \frac{144}{225} \) or \( \frac{16}{25} \)

If two similar polygons or circles have lengths of corresponding sides (or radii) in the ratio of \( \frac{a}{b} \), then their areas are in the ratio of \( \frac{a^2}{b^2} \) or \( \left( \frac{a}{b} \right)^2 \).

b. perimeter ratio: \( \frac{45}{15} \) or \( \frac{3}{1} \)  
   area ratio: \( \frac{2025}{225} \) or \( \frac{9}{1} \)

If two similar polygons or circles have lengths of corresponding sides (or radii) in the ratio of \( \frac{a}{b} \), then their areas are in the ratio of \( \frac{a^2}{b^2} \) or \( \left( \frac{a}{b} \right)^2 \).

4. a. Dimensions: \( \frac{x}{40} = \frac{3}{2} \)  
   \( 2x = 120 \)  
   \( x = 60 \)

   \( \frac{x}{20} = \frac{3}{2} \)  
   \( 2x = 60 \)  
   \( x = 30 \)

60 yards by 30 yards for fence

b. Total fence needed: \( 120 + 60 = 180 \) yards

c. Cost: \( (180 \text{ yards})(\$7.50 \text{ per yard}) = \$1,350.00 \)

5. a. 

   Scale: 
   \( \frac{1}{4} \) inch = 1 foot

   Perimeter of the deck is 30 ft.
Development of Ideas (Continued)

Answers for Activity Two (Continued)

b. \(8(7) - 3(4) = 44 \text{ ft.}^2\)

c. 2:1 The ratio of the perimeters is equal to the ratio of the corresponding sides.

d. 4:1 The ratio of the areas is equal to the square of the ratio of the corresponding sides.

6. The ratio of the areas is \(\left(\frac{16}{12}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}\)

\[\frac{16}{9} = \frac{440}{x}\] so \(x\), the cost, is $247.50.

7. \(\frac{1}{25}\)

8. \(\frac{2}{5}\)

9. The ratio of sides is 3:5 so ratio of areas is \(3^2:5^2 = 9:25\).

10. a. \(\frac{25}{15}\) or \(\frac{5}{3}\). If the figures are similar then the ratios of corresponding dimensions will be the same.

b. \(\frac{25}{9}\) If two similar polygons or circles have lengths of corresponding sides (or radii) in the ratio of \(\frac{a}{b}\), then their areas are in the ratio of \(\frac{a^2}{b^2}\) or \((\frac{a}{b})^2\).

Closure

Summary question

When a salesperson showed Sam a picture, Sam said, “I need one that is the same shape but twice as big.” The salesperson returned with two pictures and said, “I wasn’t sure what you meant.” How do you think the two pictures compared to the original? Discuss why you need to be specific when you say one thing is “twice as big” as another.

Answer: One picture could have the dimensions of the original picture doubled which would mean the area of the larger picture is four times the original picture. The second picture could have twice the area of the original picture.
Area and Volume of Similar Figures

Suggested homework

Worksheet: Areas of Similar Figures

Answers:

1. a. 4:5  
   b. \( \frac{4}{5} = \frac{x}{10} \), \( x = 8 \) centimeters

2. Ratio of lengths is \( \frac{2}{3} \) so the ratio of the areas is \( \frac{4}{9} \)
   
   \( \frac{4}{9} = \frac{x}{3} \), so \( x = 1\frac{1}{3} \) gallons

3. a. 3:4  
   b. \( \frac{3}{4} = \frac{6}{x} \), \( x = 8 \) centimeters

4. Ratio of areas = \( \frac{180}{20} = \frac{9}{1} \) so the ratio of the lengths is 3:1.
   
   \( \frac{3}{1} = \frac{20}{x} \) so \( x \) is approximately 6.66 meters

5. \( \left( \frac{1.5}{1} \right)^2 = \left( \frac{2.25}{1} \right) \) is the ratio of the areas
   
   \( \frac{2.25}{1} = \frac{x}{1000} \), so the cost, \( x \), is $2250.
Area and Volume of Similar Figures

Perimeter and Area of Similar Figures

1. The rectangles below are similar.

![Rectangle Diagram]

a. What is the similarity ratio of rectangle ABCD to rectangle MNOP? Simplify your ratio.

b. What is the value of $x$? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

c. What is the perimeter of each rectangle?

d. What is $\frac{\text{Perimeter of rectABCD}}{\text{Perimeter of rectMNOP}}$? Simplify your ratio.

2. The isosceles triangles below are similar.

![Triangle Diagram]

a. What is the similarity ratio of triangle EFG to triangle HIJ? Simplify your ratio.

b. What is the value of $x$? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

c. What is the perimeter of each triangle? (Hint: use Pythagorean Theorem). Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

d. What is $\frac{\text{Perimeter of } \triangle EFG}{\text{Perimeter of } \triangle HIJ}$? Simplify your ratio.
Area and Volume of Similar Figures

Perimeter and Area of Similar Figures (Continued)

3. Circle A and circle B are similar.
   a. What is the similarity ratio of circle A to circle B?
   b. What is the circumference of each circle? Leave answers in terms of $\pi$.
   c. What is \( \frac{\text{Circumference of circle A}}{\text{Circumference of circle B}} \)? Simplify your ratio.

4. What is the relationship between the ratio of the sides of the similar figures and the ratio of their perimeters or circumferences?

5. These rectangles (from problem 1) are similar.
   a. What is the similarity ratio of rectangle ABCD to rectangle MNOP?
   b. What is the area of each rectangle?
   c. What is \( \frac{\text{Area of rectABCD}}{\text{Area of rectMNOP}} \)? Simplify your ratio.

6. These isosceles triangles (from problem 2) are similar.
   a. What is the similarity ratio of triangle EFG to triangle HIJ?
   b. What is the area of each triangle?
   c. What is \( \frac{\text{Area of } \triangle EFG}{\text{Area of } \triangle HIJ} \)? Simplify your ratio.
Area and Volume of Similar Figures

Perimeter and Area of Similar Figures (Continued)

7. Circle A and circle B (from problem 3) are similar.
   a. What is the similarity ratio of circle A to circle B?
   b. What is the area of each circle? Leave the answers in terms of $\pi$.
   c. What is $\frac{\text{Area of circle A}}{\text{Area of circle B}}$? Simplify your ratio.

8. What is the relationship between the ratio of the sides of the similar figures and the ratio of their areas?

9. Fill in the blank.

   If two similar polygons or circles have lengths of corresponding sides (or radii) in the ratio of $\frac{a}{b}$, then their areas are in the ratio of _______.


Area and Volume of Similar Figures

Area Ratios of Similar Figures

1. a. On a piece of graph paper draw a rectangle and determine its area.

b. Draw another rectangle by doubling the length of each side of the first rectangle. Determine the area of this rectangle.

c. Explain the relationship between the areas.

2. The rectangles drawn are similar. The lengths of a pair of corresponding sides are given.

   7  

   3

   a. What is the similarity ratio of the rectangle on the left to the rectangle on the right?

   b. What is the ratio of the perimeter of the rectangle on the right to the perimeter of the rectangle on the left?

   c. What is the ratio of the area of the figure on the right to the area of the figure on the left?

3. Find the ratio of the perimeter of the figure on the left to the figure on the right. Then find the ratio of the area of the figure on the left to the figure on the right. Use mathematics to justify your answers.

   a. Regular Octagons

      12

      15

   b. Equilateral Triangles

      45

      15
Area and Volume of Similar Figures

Area Ratios of Similar Figures (Continued)

4. The dimensions of the rectangular pool shown below are 40 yards by 20 yards. Fencing is to be ordered to enclose the deck. The ratio of the dimensions of the region that is fenced in to the dimensions of the pool is $\frac{3}{2}$.

![Diagram of pool and deck]

a. What are the dimensions, in yards, of the region that is fenced in? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

b. How many yards of fence were purchased to enclose the deck?

c. Fencing costs $7.50 per yard. How much did it cost to purchase fencing to enclose the deck?

5. A scale drawing of a deck is shown below.

![Diagram of scale drawing]

a. What is the perimeter of the actual deck? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

b. What is the area of the actual deck? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

c. If the deck dimensions are doubled, what is the ratio of the perimeters of the new deck to the actual deck? Use mathematics to justify your answer.

d. If the deck dimensions are doubled, what is the ratio of the areas of the new deck to the actual deck? Use mathematics to justify your answer.
6. It costs $440 to carpet a room that measures 16 feet by 24 feet. How much would it cost to carpet a similar room that measures 12 feet by 18 feet? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

7. The lengths of corresponding altitudes in two similar triangles are in the ratio of $\frac{1}{5}$. What is the ratio of their areas?

8. The areas in two similar parallelograms are in the ratio of $\frac{4}{25}$. What is the ratio of the lengths of corresponding altitudes?

9. Given the figure at the right, what is the ratio of the areas of the small triangle to the large triangle? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

10. Two scale models of the Wright brothers plane the Kittyhawk are shown below. The two models are similar to each other because they are both similar to the original plane. The larger model is 25 cm long, and the smaller model is 15 cm long.

   a. What is the ratio of the wingspan of the larger model to the wingspan of the smaller model? Use mathematics to justify your answer.

   b. What is the ratio of the area of the top wing of the larger model to the area of the top wing of the smaller model? Use mathematics to justify your answer.
Area and Volume of Similar Figures

Areas of Similar Figures

1. The ratio of the areas of two squares is $\frac{16}{25}$.
   a. What is the ratio of their sides?
   b. The larger square has sides of length 10 centimeters. What is the side length of the smaller square? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

2. Two rooms are similar in shape, with corresponding lengths in the ratio of $\frac{2}{3}$. It takes 3 gallons of paint to cover the walls of the larger room. How much paint will be needed to paint the smaller room? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

3. The ratio of the areas of two circles is $\frac{9}{16}$.
   a. What is the ratio of their radii?
   b. The smaller circle has a radius of 6 centimeters. What is the radius of the larger circle? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

4. A rectangular yard has an area of 180 m². A similar yard has an area of 20 m². If the length of the larger yard is 20 m, what is the length of the smaller yard? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

5. Jose bought carpeting for his living room and for his dining room. His living room is similar to his dining room and 1.5 times as long. If it costs $1000 for the carpet for the dining room, how much should it have cost to buy the carpet for the living room? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.
Area and Volume of Similar Figures

Lesson Plan – Surface Area and Volume of Similar Solids

Essential Question

What is the relationship between perimeter, circumference, area, surface area, and/or volume of similar figures and their parts?

Warm-Up/Opening Activity

Recall the relationship between scale factor and corresponding dimensions using surface area ratios of similar figures.

Worksheet: Surface Area of Similar Solids

Answers:

1. b. Cube | Area-1 Face | Surface area
   A 1 cm² | 6 cm²
   B 4 cm² | 24 cm²
   C 9 cm² | 54 cm²

c. Comparing | Ratio of edge lengths | Ratio of surface areas
   A to B | 1:2 | 6:24 or 1:4
   A to C | 1:3 | 6:54 or 1:9
   B to C | 2:3 | 24:54 or 4:9

d. The ratio of the surface areas of the similar cubes equals the square of the ratio of the edges.

2. a. 4 by 6 by 8

b. Face | Area of face on prism 1 | Area of face on prism 2 | Area ratio of prism 1:prism 2
   top | 12 | 48 | 12:48 or 1:4
   Bottom | 12 | 48 | 12:48 or 1:4
   Left front | 6 | 24 | 6:24 or 1:4
   Right back | 6 | 24 | 6:24 or 1:4
   Right front | 8 | 32 | 8:32 or 1:4
   left back | 8 | 32 | 8:32 or 1:4
   Total | 52 | 208 | 52:208 or 1:4

c. 1:2

d. The ratio of the surface areas of the similar prisms equals the square of the ratio of the edges.
Area and Volume of Similar Figures

Warm-Up/Opening Activity (Continued)

Answers to Opening Activity (Continued)

3. a. 3:1
   b. left cylinder: $4374\pi = 13741.33 \text{ cm}^2$, right cylinder: $486\pi = 1526.81 \text{ cm}^2$
   c. $4374\pi:486\pi$ or 9:1
   d. The ratio of the surface areas of the similar cylinders equals the square of the ratio of the corresponding dimensions.

4. If two similar solids have corresponding dimensions in the ratio of $\frac{a}{b}$, then their surface areas are in the ratio of $\left(\frac{a}{b}\right)^2$ or $\frac{a^2}{b^2}$.

Development of Ideas

Activity Three

Investigate the relationship between scale factor and corresponding dimensions using volume ratios of similar figures.

Worksheet: Volume of Similar Solids

Answers:

1. b. A: 1 cm$^3$, B: 8 cm$^3$, C: 27 cm$^3$
   c. 
<table>
<thead>
<tr>
<th>Comparing</th>
<th>Ratio of edge lengths</th>
<th>Ratio of volumes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A to B</td>
<td>1:2</td>
<td>1:8</td>
</tr>
<tr>
<td>A to C</td>
<td>1:3</td>
<td>1:27</td>
</tr>
<tr>
<td>B to C</td>
<td>2:3</td>
<td>8:27</td>
</tr>
</tbody>
</table>
   d. The ratio of the volumes of the similar cubes equals the cube (third power) of the ratio of the edge lengths.

2. a. 4 by 6 by 8 cm
   b. first prism: 24 cm$^3$, second prism: 192 cm$^3$
   c. 1:2
   d. 24:192 or 1:8
   e. The ratio of the volumes of the similar rectangular prisms equals the cube (third power) of the ratio of the edges lengths.

3. a. 3:1
   b. left cylinder: $39366\pi = 123671.94 \text{ cm}^3$
   right cylinder: $1458\pi = 4580.44 \text{ cm}^3$
   c. $39366\pi:1458\pi$ or 27:1
   d. The ratio of the volumes of the similar cylinders equals the cube (third power) of the ratio of the corresponding dimensions.
Development of Ideas (Continued)

Answers to Activity Three (Continued)

4. If two similar solids have corresponding dimensions in the ratio
   \[ \frac{a}{b}, \] then their volumes are in the ratio of \( \left( \frac{a}{b} \right)^3 \) or \( \frac{a^3}{b^3} \).

Activity Four

Apply surface area and volume ratios of similar figures.

Worksheet: Using Similar Solids

Answers:

1. scale factor: \( \frac{7}{5} \)
   
   ratio of surface areas: \( \frac{49}{25} \)
   
   ratio of volumes: \( \frac{343}{125} \)

2. scale factor: \( \frac{8}{12} \) or \( \frac{2}{3} \)
   
   ratio of surface areas: \( \frac{64}{144} \) or \( \frac{4}{9} \)
   
   ratio of volumes: \( \frac{512}{1728} \) or \( \frac{8}{27} \)

3. scale factor: \( \frac{12}{9} \) or \( \frac{3}{4} \)
   
   ratio of surface areas: \( \frac{144}{81} \) or \( \frac{16}{9} \)
   
   ratio of volumes: \( \frac{1728}{729} \) or \( \frac{64}{27} \)

4. scale factor: \( \frac{5}{15} \) or \( \frac{1}{3} \)
   
   ratio of surface areas: \( \frac{25}{225} \) or \( \frac{1}{9} \)
   
   ratio of volumes: \( \frac{125}{3375} \) or \( \frac{1}{27} \)

5. a. \( \frac{6}{4} = \frac{y}{10} \)
   
   \( 4y = 60 \)
   
   \( y = 15 \text{ cm} \)
Area and Volume of Similar Figures

Development of Ideas (Continued)

Answers to Activity Four (Continued)

b. scale factor is \( \frac{3}{2} \); The ratio of the volumes is the scale factor cubed so ratio of the volumes is \( \left( \frac{3}{2} \right)^3 = \frac{27}{8} \).

6. Volume ratio is \( \frac{1000}{1} \), so the side ratio is \( \sqrt[3]{\frac{1000}{1}} = \frac{10}{1} \).

\[
\frac{10}{1} = \frac{x}{8}, \text{ so } x = 80 \text{ cm}
\]

7. a. \[
\left( \frac{10}{14} \right)^2 = \frac{8.12}{c},
\]

\[100c = 1591.52\]

\[c = 15.9152 \quad \text{cost should be } $15.92\]

b. \[
\left( \frac{10}{14} \right)^2 = \frac{2}{p}
\]

\[100p = 392\]

\[p = 3.92 \quad \text{almost 4 people}\]

8. \[
\left( \frac{8}{12} \right)^3 = \frac{4}{x}
\]

\[512x = 6912\]

\[x = 13.5 \quad \text{The larger trophy will weigh about 13.5 pounds.}\]

Closure

Summary questions

Draw two spheres such that the ratio of their volumes is 1:64.

Answer: The radii of the spheres should have a ratio of 1:4

Justify why all cubes are similar to each other.

Answer: The dimensions of a cube are all equal. The ratio of any corresponding dimensions of any 2 cubes will be the same ratio of the other corresponding dimensions.
Area and Volume of Similar Figures

Surface Area of Similar Solids

1. a. Build three cubes using the centimeter cubes. The cubes should have the following dimensions: Cube A 1×1×1, Cube B 2×2×2, and Cube C 3×3×3.

b. Determine the area of one face of each cube. Then calculate the total surface area of each cube.

c. Fill in the table:

<table>
<thead>
<tr>
<th>Comparing</th>
<th>Ratio of edge lengths</th>
<th>Ratio of surface areas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube A to Cube B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cube A to Cube C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cube B to Cube C</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d. How does the ratio of surface areas compare to the ratio of edge lengths?

2. Draw a rectangular prism with dimensions 2 cm by 3 cm by 4 cm resting on its largest face on the isometric grid paper on the next page. Draw a second prism that has dimensions that are double the dimensions of the first prism and should also rest on its largest face.

a. What are the dimensions of the second prism?

b. Fill in the table below with the areas of the corresponding faces.

<table>
<thead>
<tr>
<th>Face</th>
<th>Area of face on prism 1</th>
<th>Area of face on prism 2</th>
<th>Area Ratio of prism 1 to prism 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottom</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Left front</td>
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<td></td>
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<tr>
<td>Right back</td>
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<tr>
<td>Right front</td>
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<tr>
<td>Left back</td>
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<td></td>
</tr>
<tr>
<td>Total Surface Area</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. What is the similarity ratio of corresponding lengths of the similar prisms?

d. How does the ratio of surface areas compare to the ratio of edge lengths of the prisms?
Area and Volume of Similar Figures

Surface Area of Similar Solids (Continued)
3. The cylinders below are similar.

Note: Figures are not drawn to scale.

a. What is the similarity ratio of the dimensions of the cylinder on the left to the cylinder on the right?

b. What is the surface areas of each cylinder. Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

left cylinder: \[ \frac{27 \text{ cm}}{9 \text{ cm}} \]
right cylinder: \[ \frac{54 \text{ cm}}{18 \text{ cm}} \]

left cylinder: \[ \frac{54 \text{ cm}}{18 \text{ cm}} \]
right cylinder: \[ \frac{27 \text{ cm}}{9 \text{ cm}} \]

c. What is the ratio of the surface area of the left cylinder to the surface area of the right cylinder?

d. How does the ratio of surface areas compare to the similarity ratio of the dimensions of the cylinders?

4. Fill-in the blanks:

If two similar solids have corresponding dimensions in the ratio of \( \frac{a}{b} \), then their surface areas are in the ratio of \( \frac{a^2}{b^2} \).
Area and Volume of Similar Figures

Volume of Similar Solids

1. a. Build three cubes using the centimeter cubes. The cubes should have the following dimensions:
   Cube A $1 \times 1 \times 1$, Cube B $2 \times 2 \times 2$, and Cube C $3 \times 3 \times 3$.
   b. Determine the volume of each cube.
   c. Fill in the table below:

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d. How does the ratio of volumes compare to the ratio of edge lengths?

2. Draw a rectangular prism 2 cm by 3 cm by 4 cm resting on its largest face on the isometric grid paper on the next page. Draw a second prism that has dimensions that are double the dimensions of the first prism and should also rest on its largest face.
   a. What are the dimensions of the second prism?
   b. What are the volumes of the prisms?
   c. What is the similarity ratio of corresponding lengths of the similar prisms?
   d. What is the ratio of the volumes of the similar prisms?
   e. How does the ratio of volumes compare to the ratio of edge lengths of the prisms?
Area and Volume of Similar Figures

Volume of Similar Solids (Continued)
Area and Volume of Similar Figures

Volume of Similar Solids (Continued)

3. The cylinders below are similar.

Note: Figures are not drawn to scale.

![Cylinders with dimensions](image)

a. What is the similarity ratio of the dimensions of the cylinder on the left to the cylinder on the right?

b. Determine the volume of each cylinder. Use mathematics to explain how you determined your answers. Use words, symbols, or both in your explanation.

left cylinder: right cylinder:

left cylinder:  
right cylinder:

c. What is the ratio of the volume of the left cylinder to the volume of the right cylinder?

d. How does the ratio of volumes compare to the similarity ratio of the dimensions of the cylinders?

4. Fill-in the blanks:

If two similar solids have corresponding dimensions in the ratio of \( \frac{a}{b} \), then their volumes are in the ratio of ______________.
Area and Volume of Similar Figures

Using Similar Solids

For each pair of similar solids, find the scale factor of the solid on the left to the solid on the right. Then find the ratios of the surface areas and the ratio of the volumes.

1. 

scale factor:

ratio of surface areas:

ratio of volumes:

2. 

scale factor:

ratio of surface areas:

ratio of volumes:

3. 

scale factor:

ratio of surface areas:

ratio of volumes:

4. 

scale factor:

ratio of surface areas:

ratio of volumes:
Area and Volume of Similar Figures

Using Similar Solids (Continued)

5. Look at the two cylinders shown below. The ratio of corresponding diameters is equal to the ratio of the corresponding heights.

![Diagram of two cylinders](image)

a. What is the height of the large cylinder? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

b. Find the ratio of the volumes of the cylinders. Use mathematics to justify your answer.

6. The ratio of the volumes of two tetrahedrons is 1000:1. The smaller tetrahedron has a side length of 8 centimeters. What is the side length of the larger tetrahedron? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

7. Suppose that all pizzas have the same thickness and the cost and number of servings both depend only on the surface area. A pizza 10 inches in diameter costs $8.12 and serves 2 people.

a. How much should a 14-inch pizza cost? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

b. How many people would the 14-inch pizza serve? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

8. A trophy that is 8 inches tall weighs 4 pounds. A trophy of similar shape is 12 inches tall. How much does the larger trophy weigh? Assume that the weight is proportional to the volume in any solid. Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.