

Different Methods of Proof

Objectives

Students will be able to identify and apply properties of equality and properties of congruence, perform a variety of methods for organizing deductive arguments, and validate properties of geometric figures and explain the processes used.

Core Learning Goals

2.2.3 The student will use inductive or deductive reasoning.

Materials Needed

Worksheets and overheads

Pre-requisite Concepts Needed

Students will need to be able to calculate distance, slope, and midpoint on a coordinate plane and interpret and analyze conditional statements.

Approximate Time

Six 50-minute lessons or three to four 90-minute lessons

Different Methods of Proof

Lesson Plan – Justification in Proof

Essential Questions

How can deductive reasoning be used to validate conjectures?

What methods can be used to organize a deductive argument?

Warm-Up/Opening Activity

Solve a two-step equation.

Development of Ideas

Overhead: **What is a Proof?**

Justify the steps to solve an equation using the properties of equality.

Identify and apply the properties of equality and the properties of congruence to geometric figures.

Worksheet: **Justification Using Properties of Equality and Congruence**

- Answers:
1. Transitive Property
 2. Transitive Property of Congruence
 3. Subtraction Property
 4. Transitive Property
 5. Division Property
 6. Addition Property
 7. Transitive Property of Congruence
 8. Reflexive Property of Congruence
 9. Substitution Property
 10. Symmetric Property of Congruence
 11.
 1. Given
 2. Additive Property
 3. Subtractive Property
 4. Division Property
 12.
 1. Given
 2. Given
 3. Substitution Property
 4. Subtraction Property

Different Methods of Proof

Development of Ideas (Continued)

Answers to Justification Using Properties of Equality & Congruence (Continued)

13.
 1. Given
 2. Given
 3. Subtraction Property
 4. Given
 5. Given
 6. Subtraction Property
 7. Transitive Property
14.
 1. Given
 2. Given
 3. Transitive Property
 4. Reflexive Property
 5. Subtraction Property

Closure

Compare and contrast inductive and deductive reasoning.

Worksheet: **Comparison of Inductive and Deductive Reasoning**

Different Methods of Proof

What is a Proof?

A **proof** is a convincing argument that something is true. In mathematics, a proof starts with things that are agreed upon, called postulates or axioms, and then uses logic to reach a conclusion.

Conclusions are often reached in geometry by observing data and looking for patterns. This type of reasoning is called **inductive reasoning**. The conclusion reached by inductive reasoning is called a **conjecture**.

A proof in geometry consists of a sequence of statements, each supported by a reason, that starts with a given set of premises and leads to a valid conclusion. This type of reasoning is called **deductive reasoning**. Each statement in a proof follows from one or more of the previous statements. A reason for a statement can come from the set of given premises or from one of the four types of other premises: definitions; postulates; properties of algebra, equality, or congruence; or previously proven theorems. Once a conjecture is proved, it is called a **theorem**. As a theorem, it becomes a premise for geometric arguments you can use to prove other conjectures. The four common methods of geometric proofs are: 1) two-column proofs, 2) paragraph proofs, 3) flow chart proofs, and 4) coordinate proofs.

Different Methods of Proof

Justification Using Properties of Equality and Congruence

PROPERTIES OF EQUALITY FOR REAL NUMBERS	
Reflexive Property	For any number a , $a = a$.
Symmetric Property	For any numbers a and b , if $a = b$ then $b = a$.
Transitive Property	For any numbers a , b and c , if $a = b$ and $b = c$, then $a = c$.
Addition and Subtraction Properties	For any numbers a , b and c , if $a = b$, then $a + c = b + c$ and $a - c = b - c$.
Multiplication and Division Properties	For any numbers a , b and c , if $a = b$, then $a \times c = b \times c$ and if $c \neq 0$, then $a \div c = b \div c$.
Substitution Property	For any numbers a and b , if $a = b$, then a may be replaced with b in any equation.

PROPERTIES OF CONGRUENCE	
Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$
Symmetric Property of Congruence	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$
Transitive Property of Congruence	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$

Different Methods of Proof

Justification Using Properties of Equality and Congruence (Continued)

State the property of equality or property of congruence that justifies each conclusion.

- Given: $m\angle 1 = m\angle 2$
 $m\angle 2 = 75$
Conclusion: $m\angle 1 = 75$
- Given: $\overline{EG} \cong \overline{FG}$
 $\overline{FG} \cong \overline{GH}$
Conclusion: $\overline{EG} \cong \overline{GH}$
- Given: $x + 9 = 13$
Conclusion: $x = 4$
- Given: $JK = KL$
 $MN = KL$
Conclusion: $JK = MN$
- Given: $7x = 63$
Conclusion: $x = 9$
- Given: $m\angle 3 = 65$
 $m\angle 4 = 65$
Conclusion: $m\angle 3 + m\angle 4 = 130$
- Given: $\angle 1 \cong \angle 2$
 $\angle 2 \cong \angle 3$
Conclusion: $\angle 1 \cong \angle 3$
- Given: \overline{XY} is a segment
Conclusion: $\overline{XY} \cong \overline{XY}$
- Given: $2x + y = 70$
 $y = 3x$
Conclusion: $2x + 3x = 70$
- Given: $\angle A \cong \angle B$
Conclusion: $\angle B \cong \angle A$

Different Methods of Proof

Justification Using Properties of Equality and Congruence (Continued)

Supply the missing reasons for each of the following:

11.

Given: $15y + 7 = 12 - 20y$

Conclusion: $y = \frac{1}{7}$

Statement	Reason
1. $15y + 7 = 12 - 20y$	1.
2. $35y + 7 = 12$	2.
3. $35y = 5$	3.
4. $y = \frac{1}{7}$	4.

12.

Given: $m\angle 1 + m\angle 2 = 100$

$m\angle 1 = 80$

Conclusion: $m\angle 2 = 20$



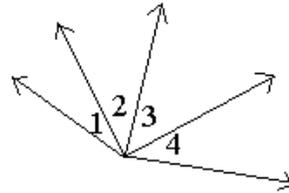
Statement	Reason
1. $m\angle 1 + m\angle 2 = 100$	1.
2. $m\angle 1 = 80$	2.
3. $80 + m\angle 2 = 100$	3.
4. $m\angle 2 = 20$	4.

Different Methods of Proof

Justification Using Properties of Equality and Congruence (Continued)

13.

Given: $m\angle 1 = 40$
 $m\angle 2 = 40$
 $m\angle 1 + m\angle 3 = 80$
 $m\angle 4 + m\angle 2 = 80$

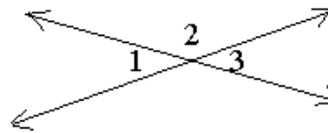


Conclusion: $m\angle 3 = m\angle 4$

Statement	Reason
1. $m\angle 1 + m\angle 3 = 80$	1.
2. $m\angle 1 = 40$	2.
3. $m\angle 3 = 40$	3.
4. $m\angle 4 + m\angle 2 = 80$	4.
5. $m\angle 2 = 40$	5.
6. $m\angle 4 = 40$	6.
7. $m\angle 3 = m\angle 4$	7.

14.

Given: $m\angle 1 + m\angle 2 = 180$
 $m\angle 2 + m\angle 3 = 180$



Conclusion: $m\angle 1 = m\angle 3$

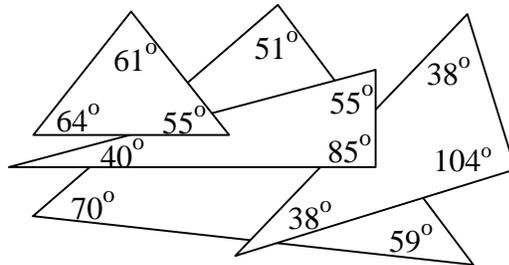
Statement	Reason
1. $m\angle 1 + m\angle 2 = 180$	1.
2. $m\angle 2 + m\angle 3 = 180$	2.
3. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	3.
4. $m\angle 2 = m\angle 2$	4.
5. $m\angle 1 = m\angle 3$	5.

Different Methods of Proof

Comparison of Inductive and Deductive Reasoning

Inductive Reasoning Process:

Observations:

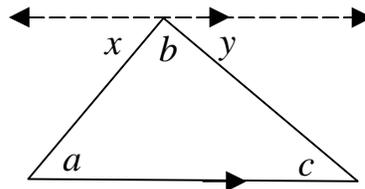


$$\begin{aligned}64^\circ + 61^\circ + 55^\circ &= 180^\circ \\70^\circ + 59^\circ + 51^\circ &= 180^\circ \\40^\circ + 55^\circ + 85^\circ &= 180^\circ \\38^\circ + 38^\circ + 104^\circ &= 180^\circ\end{aligned}$$

Generalizations:

Conjecture: $a + b + c = 180^\circ$
for all triangles!

Deductive Reasoning Process:



Facts accepted as true:

- Fact 1:** $x + b + y = 180^\circ$ because x , b , and y are measures of angles that form a straight angle.
- Fact 2:** $x = a$ and $y = c$ because alternate interior angles are congruent when parallel lines are cut by a transversal.
- Fact 3:** We can substitute equal values for equal values.

Logical consequences:

Conclusion: $a + b + c = 180^\circ$ for any triangle.

Different Methods of Proof

Lesson Plan – Flow Chart Proofs

Essential Questions

How can deductive reasoning be used to validate conjectures?

What methods can be used to organize a deductive argument?

Warm-Up/Opening Activity

Write the directions for making a sandwich in a flow chart.

Teacher Note: *You might want to have the ingredients to make a sandwich available in class.*

Development of Ideas

Arrange in order the steps to solve an algebraic equation.

Teacher Note: *Have the students work in groups for problems 5, 6, and 7. Each group of students will need an envelope with the statements and reasons for each problem cut into strips and a copy of the flowchart template for each problem. The master copies follow the worksheet.*

Justify the steps to solve the algebraic equation in a flow chart format.

Worksheet: Introduction to Flow Chart Proofs

- Answers:
- $\boxed{3x-15 = 150} \rightarrow \boxed{3x = 165} \rightarrow \boxed{x = 55}$
Given Equation Addition Property of Equality Division Property of Equality
 - d.-e. The first box is the 'if' statement and the last box is the 'then' statement.
 - Given \rightarrow Multiplication Property of Equality \rightarrow Division Property of Equality
 - b.-c. The first box is the 'if' statement and the last box is the 'then' statement.
 - $\boxed{3x+28=58} \rightarrow \boxed{3x = 30} \rightarrow \boxed{x = 10}$
Given Equation Subtraction Property of Equality Division Property of Equality

Different Methods of Proof

Development of Ideas (Continued)

Answers to Introduction to Flow Chart Proofs (Continued)

9. b. Given: $\angle A$ and $\angle B$ are complementary
 $\angle B$ and $\angle C$ are complimentary
 Prove: $\angle A \cong \angle C$

c.

$\angle A$ and $\angle B$ are complementary
Given

$\angle B$ and $\angle C$ are complimentary
Given

$\angle A + \angle B = 90^\circ$
Definition of Complementary Angles

$\angle B + \angle C = 90^\circ$
Definition of Complementary Angles

$\angle A + \angle B = \angle B + \angle C$
Transitive Property of Equality

$\angle B \cong \angle B$
Reflexive Property of Congruence

$\angle A \cong \angle C$
Subtraction Property

Justify geometric properties using a flow chart.

Worksheet: Flow Chart Proofs

Answers:

1.

$\overline{AR} \cong \overline{ER}$
Given

$\overline{AC} \cong \overline{EC}$
Given

$\triangle RCE \cong \triangle RCA$
Side-side-side triangle congruence

$\angle E \cong \angle A$
Definition of congruent triangles or CPCTC

$\overline{RC} \cong \overline{RC}$
Reflexive Property of congruence

Different Methods of Proof

Development of Ideas (Continued)

Answers to Flow Chart Proofs (Continued)

2.

$\overline{SE} \cong \overline{SU}$
Given

$\angle E \cong \angle U$
Given

$\triangle SEM \cong \triangle SUO$
Angle-Side-Angle triangle congruence

$\overline{MS} \cong \overline{SO}$
Definition of congruent triangles or CPCTC

$\angle 1 \cong \angle 2$
Definition of Vertical Angles

3.

M is midpoint of \overline{AB}
Given

$\overline{AM} \cong \overline{MB}$
Definition of midpoint

M is midpoint of \overline{CD}
Given

$\overline{CM} \cong \overline{MD}$
Definition of midpoint

$\triangle AMC \cong \triangle BMD$
Side-angle-side triangle cong.

$\overline{AC} \cong \overline{BD}$
Definition of triangle cong. or CPCTC

$\angle 1 \cong \angle 2$
Vertical angles are congruent

4.

Isosceles $\triangle FGH$
Given

$\overline{FG} \cong \overline{FH}$
Definition of isosceles triangle

\overline{EF} is a median
Given

$\overline{GE} \cong \overline{HE}$
Definition of median

$\triangle GFE \cong \triangle HFE$
Side-side-side triangle cong.

$\overline{EF} \cong \overline{EF}$
Reflexive Property of congruence

Different Methods of Proof

Closure

Describe the advantages and disadvantages of writing instructions for a task in a flow chart.

Answer: An advantage of using flow charts is to be able to show different directions and logic pathways within the sequence of directions. A disadvantage is that the pathways can be confusing and difficult to see at first.

Describe how deductive reasoning is used in flow charts.

Answer: Flow charts show how deductive reasoning is developed by using the given statements, definitions, and theorems to demonstrate proofs and showing how the connections are made.

Different Methods of Proof

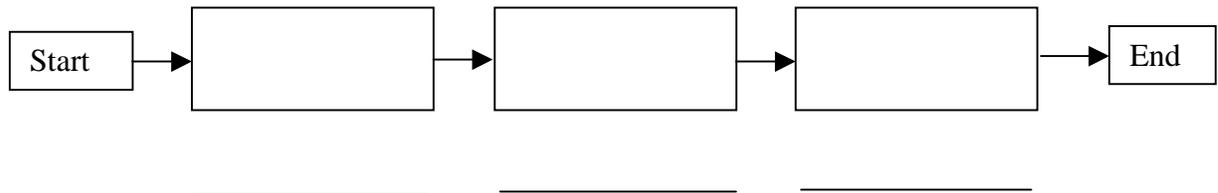
Introduction to Flow Chart Proofs

A **flow chart** proof is a concept map that shows the statements and reasons needed for a proof in a structure that helps to indicate the logical order. Statements, written in the logical order, are placed in the boxes. The reason for each statement is placed under that box.

1. a. Cut out the individual boxes of statements and reasons at the bottom of the page.
- b. Arrange the statements and reasons to prove the following conditional:

$$\text{If } 3x - 15 = 150 \text{ then } x = 55.$$

- c. Copy the statements and reasons in the proper order on the flowchart displayed below. Place the statements in the boxes and the reasons on the lines below the boxes.



- d. What is the statement in the first box? How does it relate to the conditional?
- e. What is the statement in the last box? How does it relate to the conditional?

Cut out:

$x = 55$	Addition Property of Equality	$3x = 165$
Division Property of Equality	$3x - 15 = 150$	Given equation

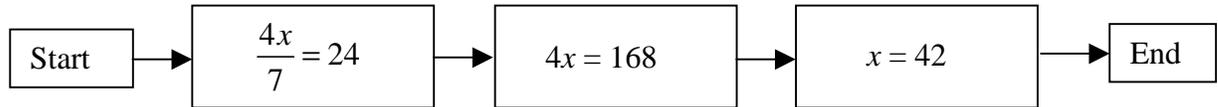
Different Methods of Proof

Introduction to Flow Chart Proofs (Continued)

2. Prove the following conditional:

$$\text{If } \frac{4x}{7} = 24, \text{ then } x = 42.$$

- a. The statements are already entered into the flowchart. Write the correct reasons below each box.



- b. What is the statement in the first box? How does it relate to the conditional?
 c. What is the statement in the last box? How does it relate to the conditional?

3. Prove the following conditional:

$$\text{If } 3x + 28 = 58, \text{ then } x = 10.$$

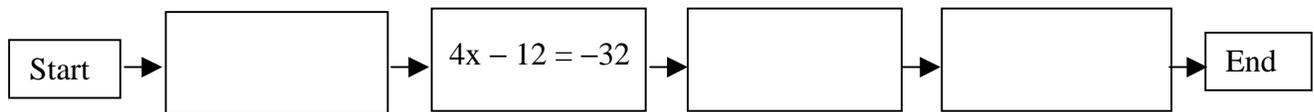
Write the correct statements and reasons in the flowchart to prove the conditional above.



4. Given the conditional:

$$\text{If } 5x - 12 = x - 32, \text{ then } x = -5.$$

Write the correct statements and reasons in the flowchart to prove the conditional above.



_____ Addition Property _____

Different Methods of Proof

Introduction to Flow Chart Proofs (Continued)

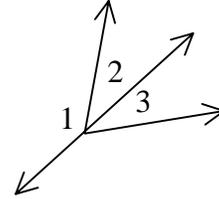
5. The flowchart proof can be used to show the logical process in a proof of a geometric idea. For example, given the following conditional:

If $\angle 1$ and $\angle 2$ are supplementary and $\angle 2 \cong \angle 3$, then $m\angle 1 + m\angle 3 = 180^\circ$.

- a. State the given and prove for this conditional.

Given:

Prove:



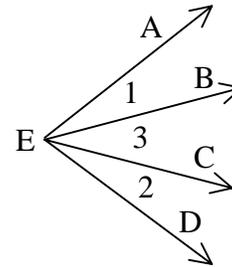
- b. Sort the slips of paper from the envelope into statements and reasons. Then arrange the statements and reasons on the flowchart to give a logical proof of the conditional.
- c. What is the statement in the first box? How does it relate to the conditional?
- d. What is the statement in the last box? How does it relate to the conditional?
6. Prove the following conditional:

If $m\angle 1 = m\angle 2$, then $m\angle AEC = m\angle BED$.

- a. State the given and prove for this conditional.

Given:

Prove:



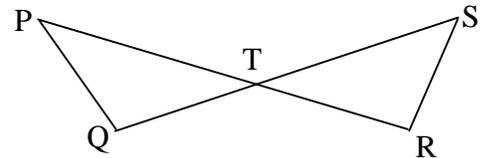
- b. Sort the slips of paper from the envelope into statements and reasons. Then arrange the statements and reasons on the flowchart to give a logical proof of the conditional.
7. Prove the following conditional:

*If \overline{PR} and \overline{QS} bisect each other at T ,
then $\triangle PQT \cong \triangle RST$.*

- a. State the given and prove for this conditional.

Given:

Prove:



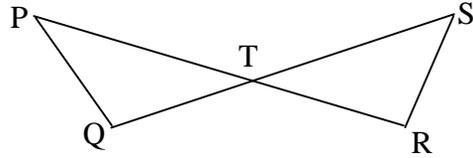
- b. Sort the slips of paper from the envelope into statements and reasons. Then arrange the statements and reasons on the flowchart to give a logical proof of the conditional.

Different Methods of Proof

Introduction to Flow Chart Proofs (Continued)

8. Prove the following conditional:

*If \overline{PR} and \overline{QS} bisect each other at T ,
then $\angle P \cong \angle R$.*



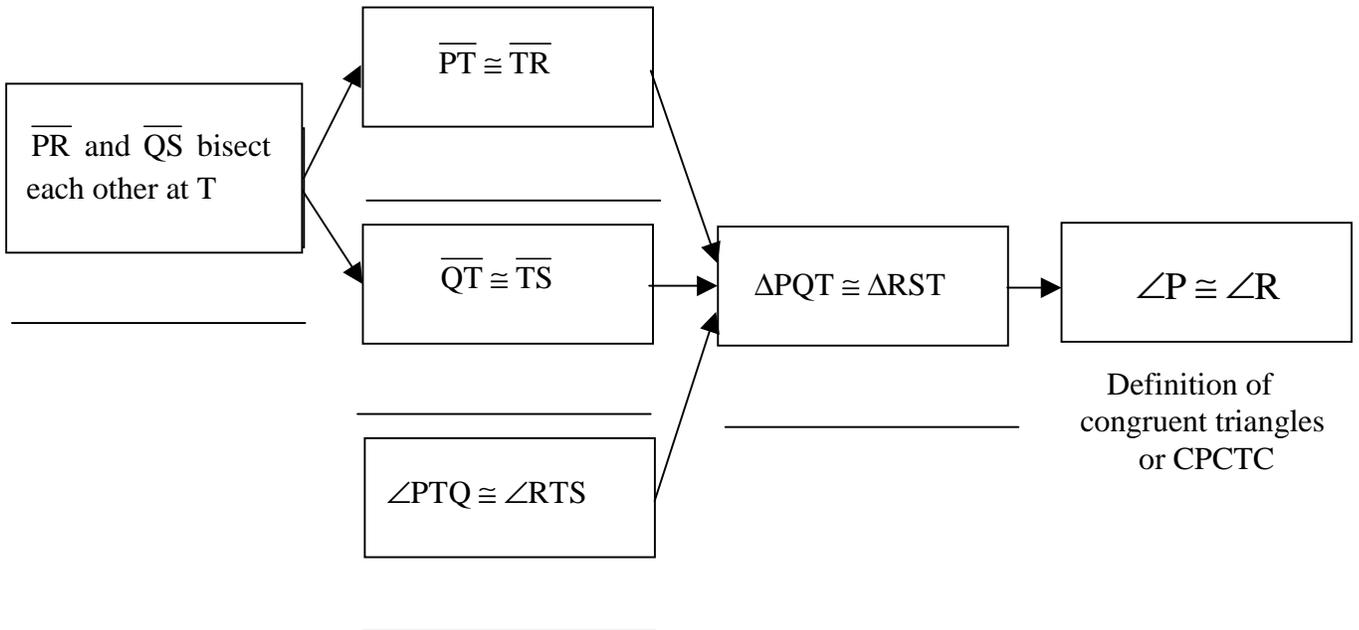
a. Complete the following:

Given:

Prove:

b. Mark the information that is given on the diagram.

c. Complete the missing parts of the flow chart proof.



Different Methods of Proof

Introduction to Flow Chart Proofs (Continued)

9. If $\angle A$ and $\angle B$ are complementary and $\angle B$ and $\angle C$ are complimentary, then $\angle A \cong \angle C$.

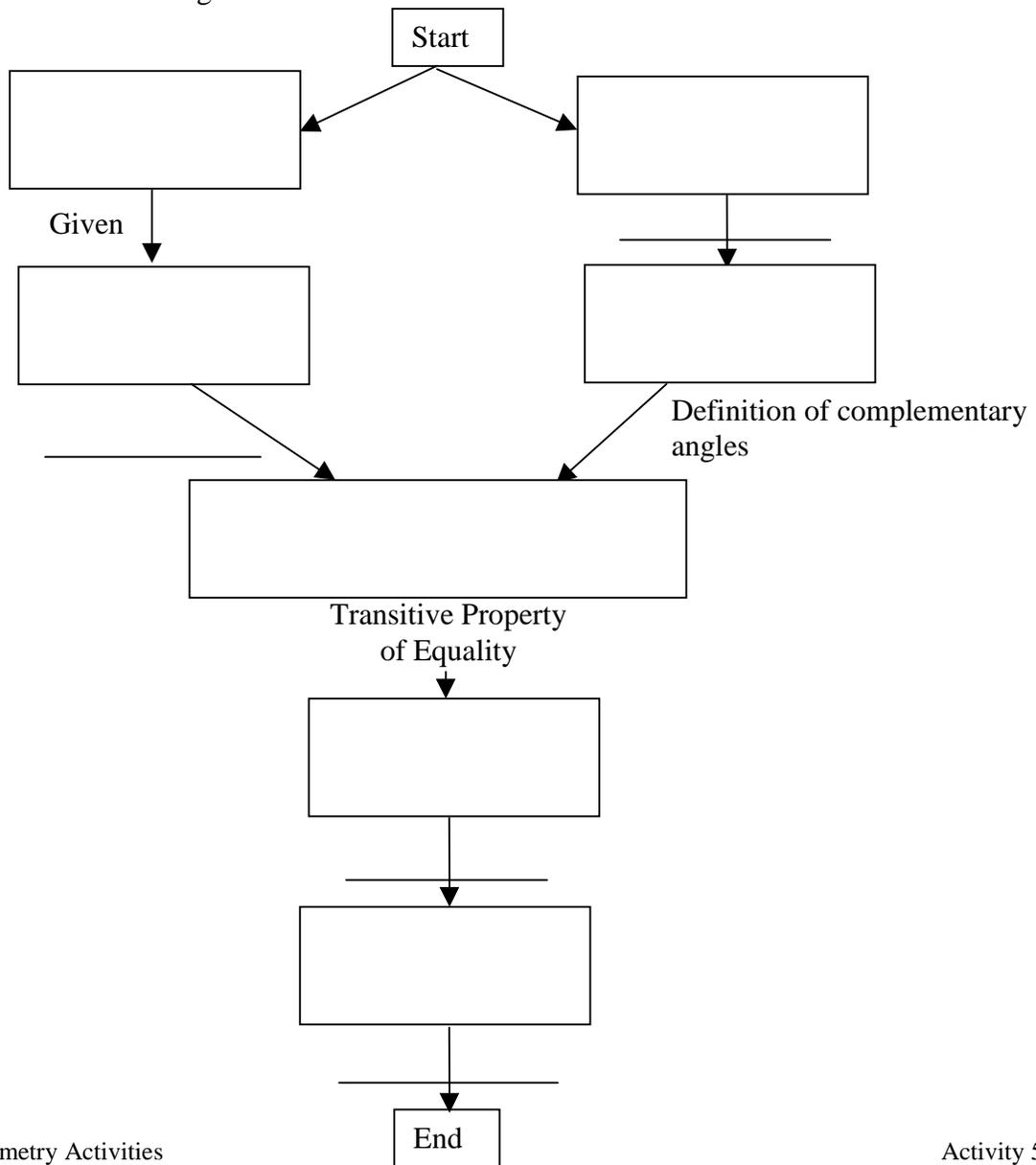
a. Draw a diagram for this conditional.

b. State the given and prove for this conditional in terms of the diagram.

Given:

Prove:

c. Fill in the missing reasons in the flowchart below.



Different Methods of Proof

Introduction to Flow Chart Proofs (Continued)

Statements and Reasons for problem 5 flowchart proof

$\angle 1$ and $\angle 2$ are supplementary	Given
$m\angle 1 + m\angle 3 = 180^\circ$	Definition of congruent angles
$m\angle 1 + m\angle 2 = 180^\circ$	Substitution property of equality
$\angle 2 \cong \angle 3$	Definition of supplementary angles
$m\angle 2 = m\angle 3$	Given

Statements and Reasons for problem 6 flowchart proof

$m\angle 1 = m\angle 2$	Angle addition postulate
$m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	Substitution property of equality
$m\angle AEC = m\angle 1 + m\angle 3$ $m\angle BED = m\angle 2 + m\angle 3$	Given
$m\angle AEC = m\angle BED$	Addition property of equality

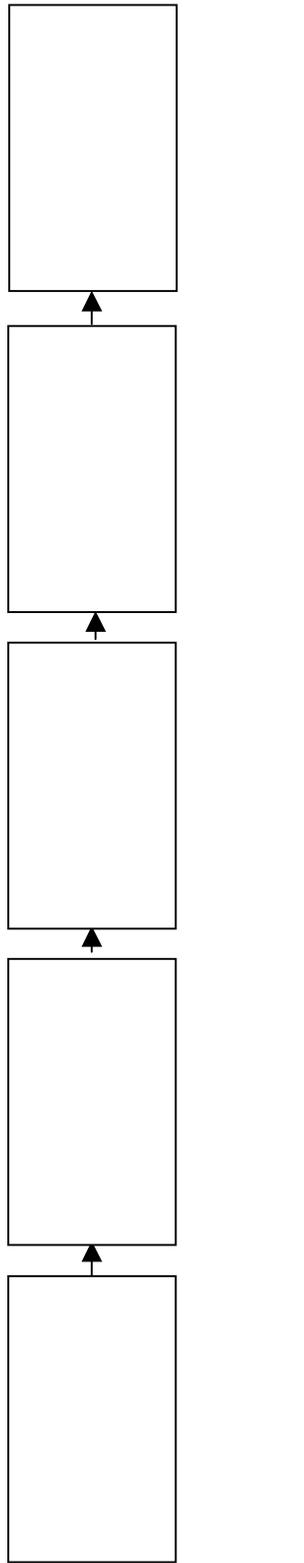
Statements and Reasons for problem 7 flowchart proof

\overline{PR} and \overline{QS} bisect each other at T	Given
$\overline{PT} \cong \overline{TR}$	Vertical angles are congruent
$\overline{QT} \cong \overline{TS}$	Definition of bisector
$\angle PTQ \cong \angle RTS$	Side-Angle-Side Congruence
$\triangle PQT \cong \triangle RST$	Definition of bisector

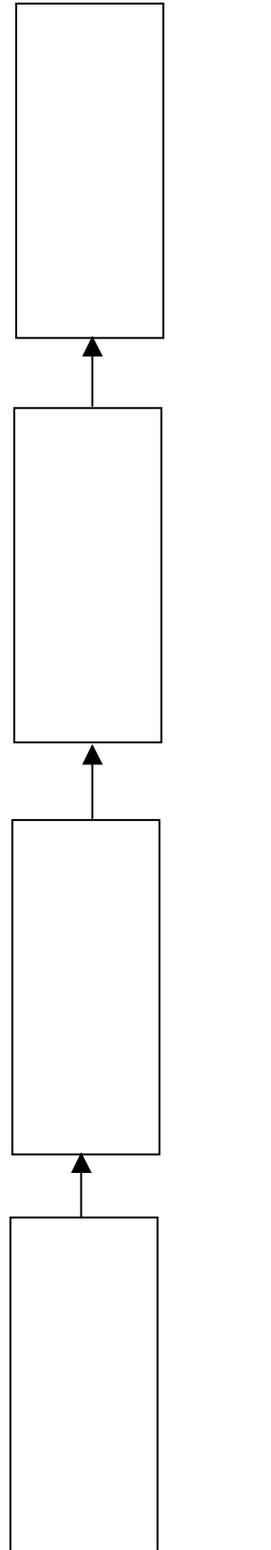
Different Methods of Proof

Introduction to Flow Chart Proofs (Continued)

Flowchart for problem 5

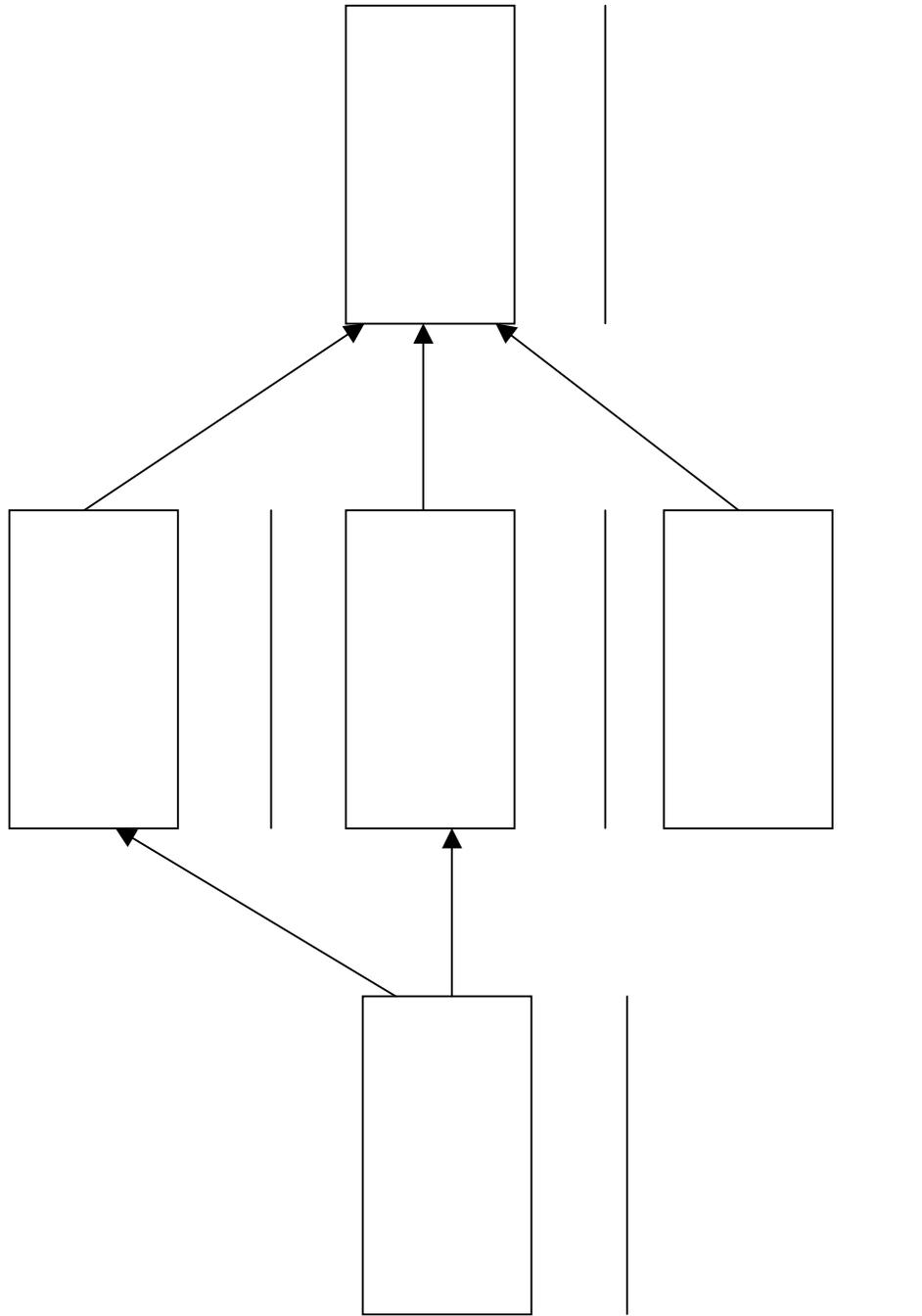


Flowchart for problem 6



Different Methods of Proof

Introduction to Flow Chart Proofs (Continued)



Flowchart for problem 7

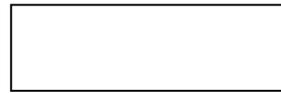
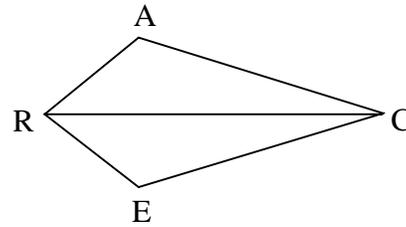
Different Methods of Proof

Flow Chart Proofs

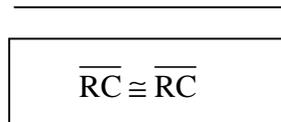
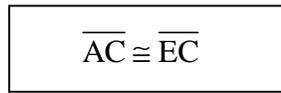
Fill in the missing reasons or statements in each proof.

1. Given: $\overline{AR} \cong \overline{ER}$
 $\overline{AC} \cong \overline{EC}$

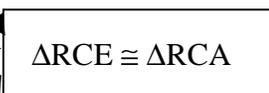
Prove: $\angle E \cong \angle A$



Given



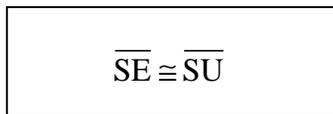
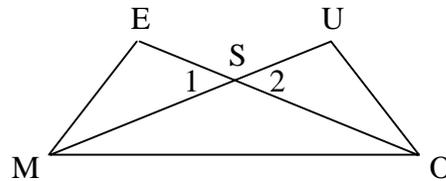
Reflexive property
of congruence



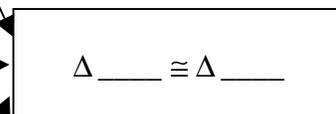
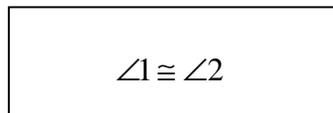
Definition of
congruent triangles
or CPCTC

2. Given: $\overline{SE} \cong \overline{SU}$
 $\angle E \cong \angle U$

Prove: $\overline{MS} \cong \overline{SO}$



Given



Angle-Side-Angle
congruence

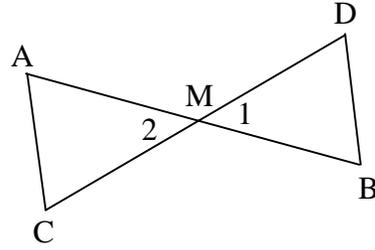


Definition of
congruent triangles
or CPCTC

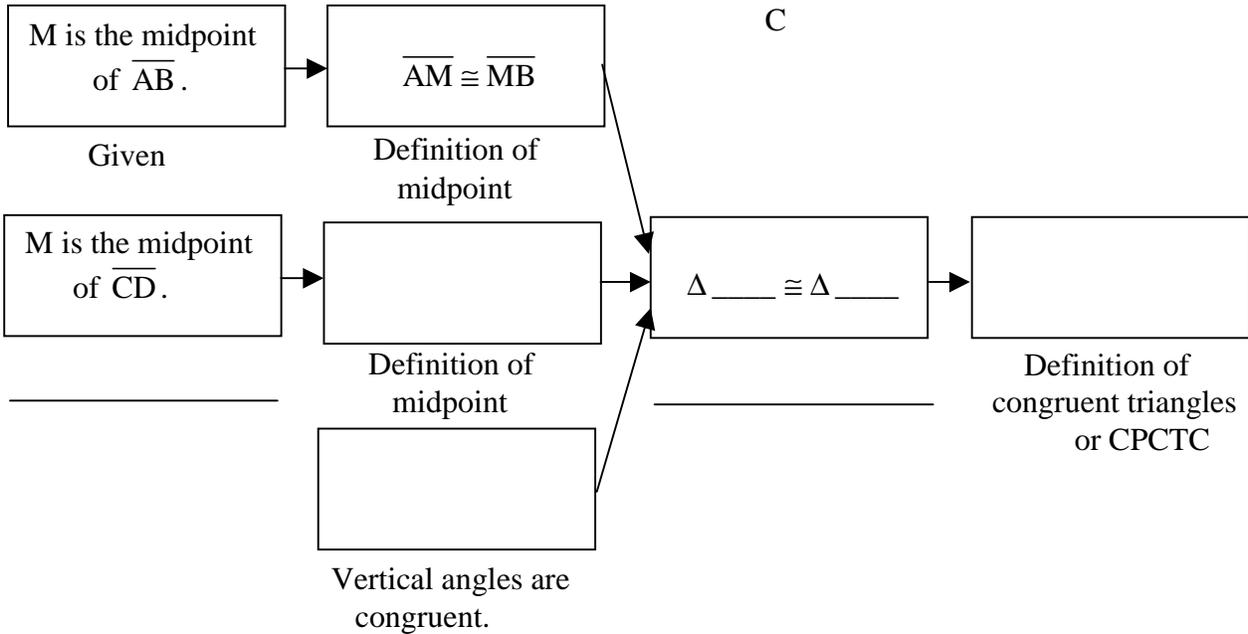
Different Methods of Proof

Flow Chart Proofs (Continued)

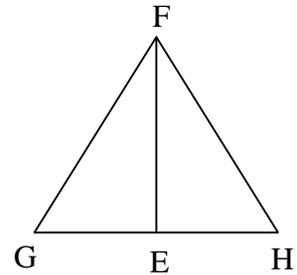
3. Given: M is the midpoint of \overline{AB} .
 M is the midpoint of \overline{CD} .



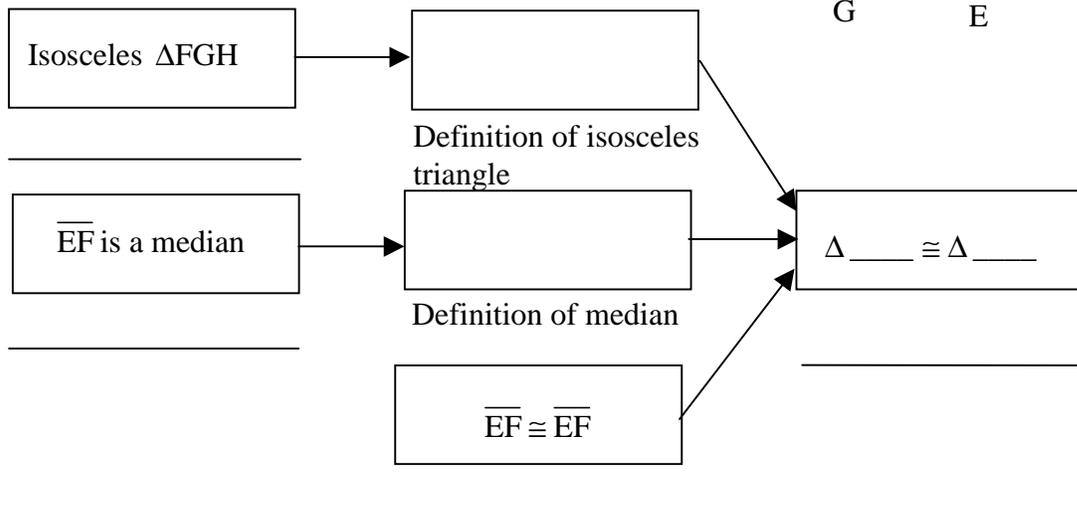
Prove: $\overline{AC} \cong \overline{BD}$



4. Given: Isosceles $\triangle FGH$ with base \overline{GH}
 \overline{EF} is a median



Prove: $\triangle GFE \cong \triangle HFE$



Different Methods of Proof

Lesson Plan – Paragraph Proofs

Essential Questions

How can deductive reasoning be used to validate conjectures?

What methods can be used to organize a deductive argument?

Warm-Up/Opening Activity

Construct a flow chart demonstrating the steps taken to get ready for school.

Development of Ideas

Convert the boxes of the flowchart to sentences and form a paragraph showing the steps taken to get ready for school.

Convert flow-chart proofs to paragraph proofs.

Justify geometric properties in paragraph form.

Worksheet: **Introduction to Paragraph Proofs**

- Answers:
1.
 - a. The first sentence contains the given statements.
 - b. The last sentence contains what is to be proved.
 2. Lines \overline{AB} and \overline{CD} are parallel and E is the midpoint of \overline{AD} . Since the \overline{AB} and \overline{CD} are parallel, angles BAE and CDE are congruent because if two parallel lines are cut by a transversal, then alternate interior angles are congruent. For this same reason, angles ABE and DCE are congruent. Since E is the midpoint of \overline{AD} , \overline{AE} and \overline{ED} are congruent. Therefore, by angle-angle-side triangle congruence, $\triangle ABE$ is congruent to $\triangle DCE$.
 3. Since $\overline{DG} \parallel \overline{EF}$ and $\overline{DE} \parallel \overline{GF}$ are given, then $\angle 1 \cong \angle 4$ and $\angle 3 \cong \angle 2$ because when two parallel lines are cut by a transversal, then alternate interior angles are congruent. $\overline{DF} \cong \overline{DF}$ because of the reflexive property of congruence. Then $\triangle DGF \cong \triangle FED$ by angle-side-angle triangle congruence. Therefore, $\overline{DG} \cong \overline{EF}$ by the definition of triangle congruence.

Different Methods of Proof

Development of Ideas (Continued)

Answers to Introduction to Paragraph Proofs (Continued)

4. Since \overline{AC} bisects $\angle BAD$ is given, then $\angle BAC \cong \angle CAD$ because of the definition of angle bisectors. Since \overline{AC} bisects $\angle BCD$ is given, then $\angle BCA \cong \angle ACD$ because of the definition of angle bisectors. $\overline{AC} \cong \overline{AC}$ because of the reflexive property of congruence. Then, $\triangle BAC \cong \triangle DAC$ by the angle-side-angle triangle congruence theorem. Therefore $\overline{AB} \cong \overline{AD}$ by the definition of triangle congruence.
5. Since $\angle E$ and $\angle S$ are right angles, they both measure 90 degrees by the definition of right angles. Because of this, they are also congruent. We are also given that $\overline{EF} \cong \overline{ST}$ and $\overline{ED} \cong \overline{SR}$. Because of this information, $\triangle DEF \cong \triangle RST$ because of side-angle-side triangle congruence.

Justify geometric properties in paragraph form.

Worksheet: Paragraph Proofs

- Answers:
1. $\angle A$ is congruent to $\angle B$ and $\angle A$ is supplementary to $\angle B$. Since the two angles are supplementary, their sum is 180° . Since they are congruent, they can be substituted for one another, meaning that $\angle A + \angle B$ is equal to 180° , but also that 2 times ($\angle B$) is equal to 180° . Then, $\angle B = 90^\circ$ by the division property of equality. Since the two angles are congruent, $\angle A$ also = 90° . $\angle A$ and $\angle B$ are right angles by the definition of right angles.
 2. $\angle 1$ and $\angle 3$ are vertical angles. Since they are vertical angles, there is an angle in between them, $\angle 2$, which is adjacent to both angles and supplementary to both angles. Since both $\angle 1$ and $\angle 3$ are supplementary to $\angle 2$, $\angle 1 + \angle 2 = 180^\circ$ and $\angle 2 + \angle 3 = 180^\circ$. $\angle 1 + \angle 2 = \angle 2 + \angle 3$ by the application of the transitive property of equality. $\angle 1$ and $\angle 3$ are congruent because to the subtraction property of equality.
 3. ABCD is a rectangle with \overline{AC} and \overline{BD} as diagonals. Since ABCD is a rectangle, opposite sides \overline{AB} and \overline{CD} are congruent. In addition, \overline{BC} and \overline{DA} are congruent. Since

Different Methods of Proof

Development of Ideas (Continued)

Answers to Paragraph Proofs (Continued)

3. (cont.) ABCD is a rectangle, $\angle B$ and $\angle C$ are right angles, and both equal to 90° by the definition of right angles. Since both are equal to 90° , they are equal to one another by the transitive property. $\triangle ABC$ and $\triangle BCD$ are congruent by the side-angle-side triangle congruence theorem. \overline{AC} is congruent to \overline{BD} by the definition of congruent triangles.

4. ABCD is a rhombus with diagonals \overline{AC} and \overline{BD} . Since ABCD is a rhombus, all four sides are congruent. In addition, $\overline{AC} \cong \overline{AC}$ and $\overline{BD} \cong \overline{BD}$ by the reflexive property of congruence. $\triangle ABC$ is congruent to $\triangle CDA$ and $\triangle BCD$ is congruent to $\triangle DAB$ by the side-side-side triangle congruence theorem. Therefore,

$$\begin{array}{ll} \angle BAC \cong \angle DAC & \angle BCA \cong \angle DCA \\ \angle ABD \cong \angle CBD & \angle CDA \cong \angle ADC \end{array}$$

by the definition of triangle congruence. \overline{AC} bisects $\angle BAD$ and $\angle BCD$ and \overline{BD} bisects $\angle ADC$ and $\angle ABC$ by the definition of angle bisectors.

5. $\angle B$ is inscribed in circle O and \widehat{ABC} is a semicircle. The measure of arc ABC is 180° by the definition of a semicircle. The $m\angle B$ is 90° because the measure of an inscribed angle is half the measure of its intercepted arc. Therefore, by definition of a right angle, $\angle B$ is a right angle.

6. Quadrilateral ABCD is inscribed in circle O. There are 360° in a circle, so $m\widehat{ABC} + m\widehat{CDA} = 360^\circ$ and the $m\widehat{BCD} + m\widehat{DAB} = 360^\circ$. By the division property of

equality, $\frac{1}{2} m\widehat{ABC} + \frac{1}{2} m\widehat{CDA} = 180^\circ$ and

$$\frac{1}{2} m\widehat{BCD} + \frac{1}{2} m\widehat{DAB} = 180^\circ.$$

The $m\angle D = \frac{1}{2} m\widehat{ABC}$, $m\angle B = \frac{1}{2} m\widehat{CDA}$,

$m\angle A = \frac{1}{2} m\widehat{BCD}$, and $m\angle C = \frac{1}{2} m\widehat{DAB}$ because the

Different Methods of Proof

Development of Ideas (Continued)

Answers to Paragraph Proofs (Continued)

6. (cont.) measure of an inscribed angle is one-half the measure of its intercepted arc. $m\angle D + m\angle B = 180^\circ$ and $m\angle A + m\angle C = 180^\circ$ by substitution. Therefore, $\angle A$ is supplementary to $\angle C$ and $\angle B$ is supplementary to $\angle D$ by definition of supplementary angles.
7. \overline{AB} is parallel to \overline{CD} . Draw \overline{AD} . $\angle BAD \cong \angle CDA$ because if two parallel lines are cut by a transversal, the alternate interior angles are congruent. $\angle BAD = \angle CDA$ by the definition of congruent angles. $m\angle BAD = \frac{1}{2}m\widehat{BD}$ and $m\angle CDA = \frac{1}{2}m\widehat{AC}$ because the measure of an inscribed angle is one-half the measure of its intercepted arc. $\frac{1}{2}m\widehat{BD} = \frac{1}{2}m\widehat{AC}$ by substitution. The measure of $\widehat{BD} = m\widehat{AC}$ by the multiplication property of equality. Therefore, $\widehat{AC} \cong \widehat{BD}$ by the definition of congruent arcs.

Closure

Explain how deductive reasoning is used in paragraph proofs.

Answer: Deductive reasoning is used to connect the given statements by use of definitions, theorems, and postulates to what is to be proved.

Different Methods of Proof

Introduction to Paragraph Proofs

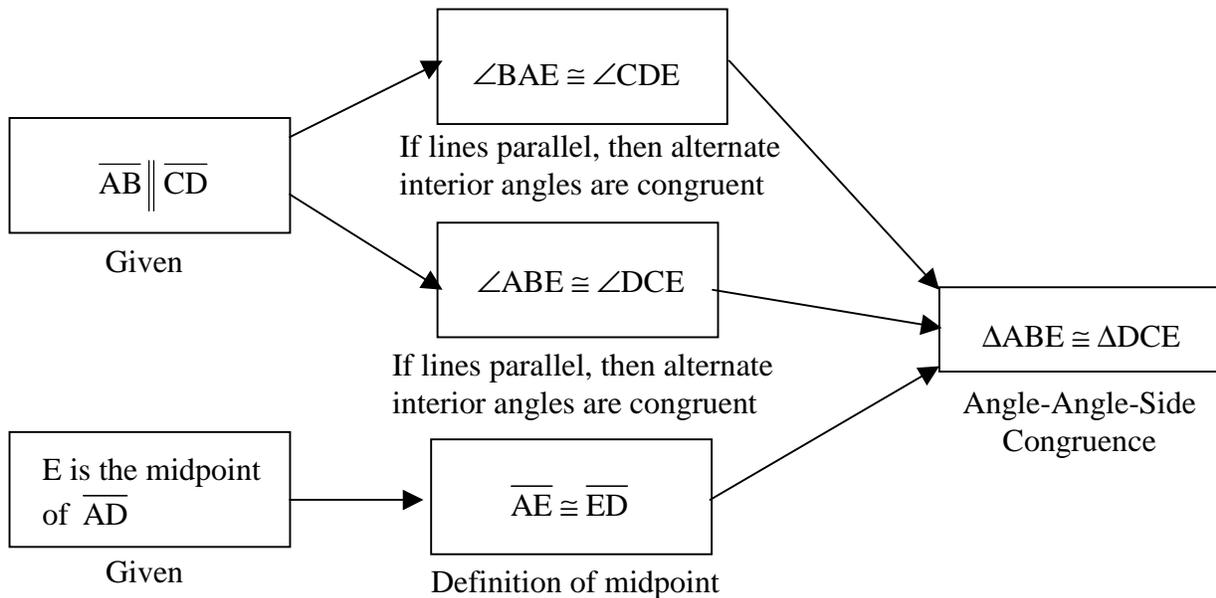
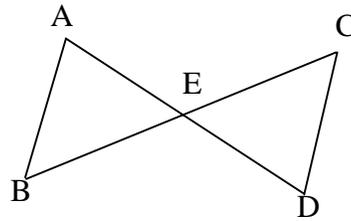
A **paragraph proof** is another way a proof is often written. The advantage of a paragraph proof is that you have the chance to explain your reasoning in your own words. In a paragraph proof, the statements and their justifications are written together in a logical order in a paragraph form. There is always a diagram and a statement of the given and prove sections before the paragraph.

1. a. What information does the first sentence of a paragraph proof contain?
- b. What information does the last sentence of a paragraph proof contain?
2. For the flow chart proof below, rewrite each box as a statement with the reason for the box as the justification.

Given: $\overline{AB} \parallel \overline{CD}$

E is the midpoint of \overline{AD}

Prove: $\triangle ABE \cong \triangle DCE$



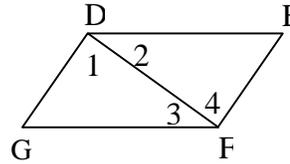
Different Methods of Proof

Introduction to Paragraph Proofs (Continued)

3. Fill-in the missing statements and justifications in the following paragraph proof.

Given: $\overline{DG} \parallel \overline{EF}$, $\overline{DE} \parallel \overline{GF}$

Prove: $\overline{DG} \cong \overline{EF}$

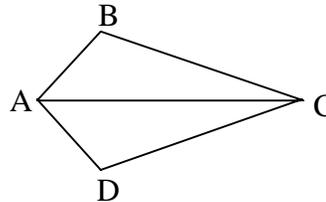


Since $\overline{DG} \parallel \overline{EF}$ and $\overline{DE} \parallel \overline{GF}$ are given, then $\angle 1 \cong \angle 4$ and $\angle 3 \cong \angle \underline{\hspace{2cm}}$ because $\underline{\hspace{2cm}}$. $\overline{DF} \cong \overline{DF}$ because $\underline{\hspace{2cm}}$. Then $\triangle DGF \cong \triangle \underline{\hspace{2cm}}$ by $\underline{\hspace{2cm}}$. Therefore, $\overline{DG} \cong \overline{EF}$ by $\underline{\hspace{2cm}}$.

4. Fill-in the missing statements and justifications in the following paragraph proof.

Given: \overline{AC} bisects $\angle BAD$
 \overline{AC} bisects $\angle BCD$

Prove: $\overline{AB} \cong \overline{AD}$

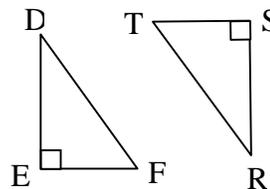


Since \overline{AC} bisects $\angle BAD$ is given, then $\angle BAC \cong \angle \underline{\hspace{2cm}}$ because $\underline{\hspace{2cm}}$. Since \overline{AC} bisects $\angle BCD$ is given, then $\angle BCA \cong \angle \underline{\hspace{2cm}}$ because $\underline{\hspace{2cm}}$. $\overline{AC} \cong \overline{AC}$ because $\underline{\hspace{2cm}}$. Then $\triangle BAC \cong \triangle \underline{\hspace{2cm}}$ by $\underline{\hspace{2cm}}$. Therefore $\overline{AB} \cong \overline{AD}$ by $\underline{\hspace{2cm}}$.

5. Mark the given on the figure. Write your own paragraph proof for the following information.

Given: $\angle E$ and $\angle S$ are right angles.
 $\overline{EF} \cong \overline{ST}$ and $\overline{ED} \cong \overline{SR}$

Prove: $\triangle DEF \cong \triangle RST$



Different Methods of Proof

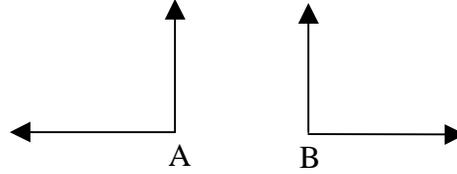
Paragraph Proofs

Use a paragraph proof to justify the following conjectures.

1. If two angles are both congruent and supplementary, then each angle is a right angle.

Given: $\angle A \cong \angle B$
 $\angle A$ is supplementary to $\angle B$

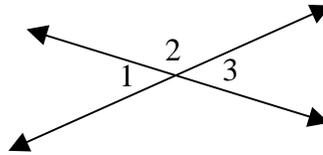
Prove: $\angle A$ is a right angle
 $\angle B$ is a right angle



2. Vertical angles are congruent.

Given: $\angle 1$ and $\angle 3$ are vertical angles

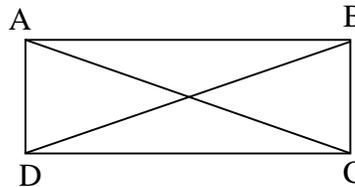
Prove: $\angle 1 \cong \angle 3$



3. The diagonals of a rectangle are congruent.

Given: Rectangle ABCD with diagonals
 \overline{AC} and \overline{BD}

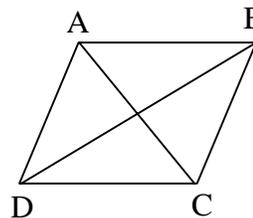
Prove: $\overline{AC} \cong \overline{BD}$



4. The diagonals of a rhombus bisect the angles.

Given: Rhombus ABCD with diagonals
 \overline{AC} and \overline{BD}

Prove: \overline{AC} bisects $\angle BAD$ and $\angle BCD$
 \overline{BD} bisects $\angle ADC$ and $\angle ABC$



Different Methods of Proof

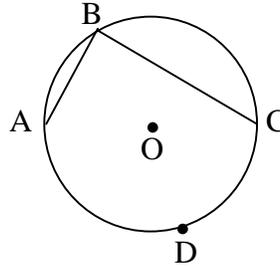
Paragraph Proofs (Continued)

5. Angles inscribed in a semicircle are right angles.

Given: $\angle B$ is inscribed in circle O

\widehat{ABC} is a semicircle

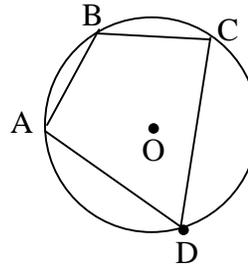
Prove: $\angle B$ is a right angle



6. If a quadrilateral is inscribed in a circle, then the opposite angles are supplementary.

Given: Quadrilateral ABCD is inscribed in circle O

Prove: $\angle A$ is supplementary to $\angle C$
 $\angle B$ is supplementary to $\angle D$

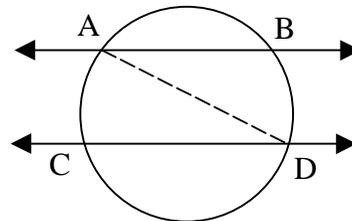


7. Parallel lines intercept congruent arcs on a circle.

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Prove: $\widehat{AC} \cong \widehat{BD}$

(Hint: Draw segment AD)



Different Methods of Proof

Lesson Plan – Two-Column Proofs

Essential Questions

How can deductive reasoning be used to validate conjectures?

What methods can be used to organize a deductive argument?

Warm-Up/Opening Activity

Develop a flow chart proof and a paragraph proof for solving an algebraic equation.

Development of Ideas

Construct a two-column proof for solving an equation.

Worksheet: **Introduction to Two-Column Proof**

- Answers: 1. a. $x = 17$
b.

$5x - 28 = 57$
Given

$5x = 85$
Addition Property

$x = 17$
Division Property

- c. We start with the equation $5x - 28 = 57$. The first step to solve this equation is to add 28 to both sides of the equation, demonstrating the addition property of equality. Next, divide both sides of the equation by 5 using the division property of equality. Therefore, x is 17.

d.

$5x - 28 = 57$	Given
$5x = 85$	Addition Property
$x = 17$	Division Property

2. a. Given: \overline{AC} and \overline{BD} bisect each other at M
Prove: $\triangle AMB \cong \triangle CMD$
c. 1. Given
2. Definition of bisector
3. Definition of bisector
4. Definition of vertical angles
5. Side-angle-side triangle congruence

Different Methods of Proof

Development of Ideas (Continued)

Answers to Introduction to Two Column Proof (Continued)

2. d.

	$\overline{AM} \cong \overline{MC}$ Definition of bisector	
\overline{AC} and \overline{BC} bisect each other Given	$\overline{BM} \cong \overline{MD}$ Definition of bisector	$\triangle AMB \cong \triangle CMD$ Side-angle-side triangle congruence
	$\angle AMB \cong \angle CMD$ Definition of vertical angles	

3. Answers will vary. Check student reasoning so be sure that it matches selection.

Justify geometric properties using a two-column proof format.

Worksheet: Two-Column Proofs

Answers:

1.
 1. Given
 2. Given
 3. Definition of angle bisector
 4. Reflexive property of congruence
 5. Side-angle-side triangle congruence
 6. Definition of congruent triangles

2.
 1. Given
 2. Given
 3. Reflexive property of congruence
 4. Side-side-side triangle congruence
 5. Definition of congruent triangles
 6. If alternate interior angles are congruent then the lines are parallel.

3.
 1. ABCD is a parallelogram
 2. $\overline{AB} \cong \overline{DC}$
 3. $\overline{AE} \cong \overline{EC}$
 4. $\overline{DE} \cong \overline{EB}$
 5. $\triangle ABE \cong \triangle CDE$

} Note: lines 3 and 4 are interchangeable

Different Methods of Proof

Development of Ideas (Continued)

Answers to Two Column Proofs (Continued)

	<u>Statement</u>	<u>Reason</u>
4.	1. $\overline{DE} \parallel \overline{AV}$	1. Given
	2. DAVE is a trapezoid	2. Definition of trapezoid
	3. $\triangle DAV \cong \triangle EVA$	3. Given
	4. $\overline{DA} \cong \overline{EV}$	4. Definition of cong. tri.
	5. DAVE is an isosceles trapezoid	5. Definition of isosceles trapezoid
5.	1. Given	
	2. Definition of a semicircle	
	3. An angle inscribed in a semicircle is a right angle	
	4. All right angles are congruent	
	5. Reflexive property of congruence	
	6. Given	
	7. Hypotenuse-Leg Congruence	

Closure

Compare and contrast flow-chart proofs and two-column proofs.

Answer: Flow-chart proofs and two-column proofs both organize statements and reasons together but flow-chart proofs allow multiple pathways and connections as where two-column proofs are always linear in reasoning.

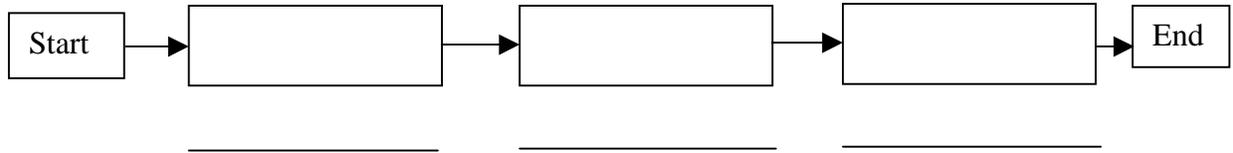
Different Methods of Proof

Introduction to Two-Column Proof

1. a. Solve the following equation.

$$5x - 28 = 57$$

- b. Draw a flowchart showing the steps and reasons for each step in solving the equation.



- c. Write a paragraph explaining the steps needed to solve the equation and justifying each step.

- d. Fill in the chart below showing the steps for solving the equation.

Statements	Reasons

A **two-column statement-reason proof** resembles the chart used to show the solution to the equation above. The statements are listed in logical order on the left side and the reason each statement is true is on the right side. The last statement is always what is being proven.

Different Methods of Proof

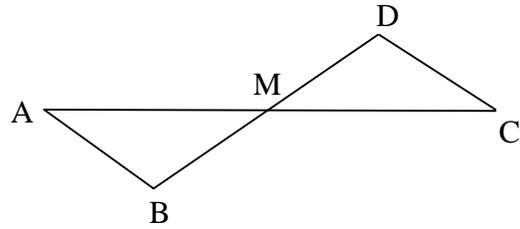
Introduction to Two-Column Proof (Continued)

2. If \overline{AC} and \overline{BD} bisect each other at M then $\triangle AMB \cong \triangle CMD$.

a. Complete the following:

Given:

Prove:



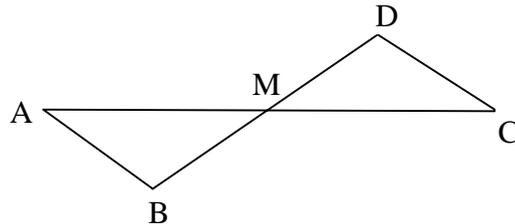
b. Mark the diagram with the given information.

The paragraph proof would be written as follows:

Since it is given that \overline{AC} and \overline{BD} bisect each other at M then $\overline{AM} \cong \overline{MC}$ and $\overline{BM} \cong \overline{MD}$ by the definition of bisect. $\angle AMB \cong \angle CMD$ since vertical angles are congruent. Therefore $\triangle AMB \cong \triangle CMD$ by Side-Angle-Side congruence.

c. This paragraph proof can be represented in a two-column statement-reason proof. The statements in logical order needed for the proof are already entered. Fill in each missing reason below.

Given:



Prove:

Statements	Reasons
1. \overline{AC} and \overline{BD} bisect each other at M	1.
2. $\overline{AM} \cong \overline{MC}$	2.
3. $\overline{BM} \cong \overline{MD}$	3.
4. $\angle AMB \cong \angle CMD$	4.
5. $\triangle AMB \cong \triangle CMD$	5.

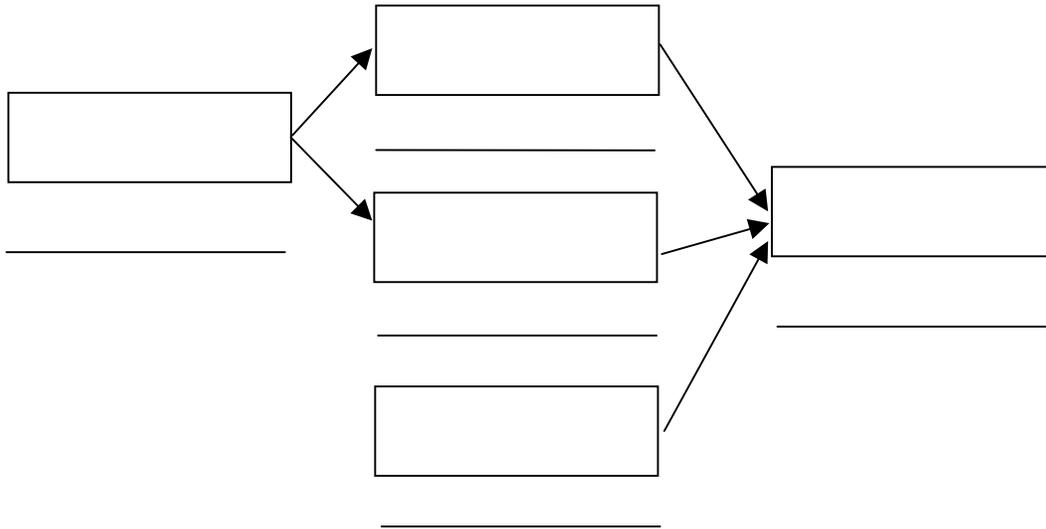
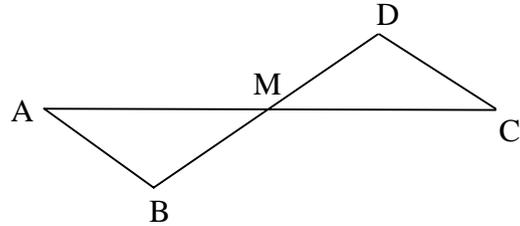
Different Methods of Proof

Introduction to Two-Column Proof (Continued)

- d. Below is the outline of a flow chart proof for the same given and prove. Fill in the boxes and reasons to complete the proof.

Given:

Prove:



3. Which of the three types of proof, flow-chart, paragraph, or two-column, is easiest for you to understand? Explain.

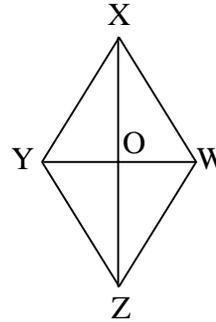
Different Methods of Proof

Two-Column Proofs

1. Mark the given information on the diagram. Give a reason for each step in the two-column proof. Choose the reason for each statement from the list below.

Given: $\overline{YX} \cong \overline{WX}$
 \overline{ZX} bisects $\angle YXW$

Prove: $\overline{YZ} \cong \overline{WZ}$



Statement	Reason
1. $\overline{YX} \cong \overline{WX}$	1.
2. \overline{ZX} bisects $\angle YXW$	2.
3. $\angle YXZ \cong \angle WXZ$	3.
4. $\overline{XZ} \cong \overline{XZ}$	4.
5. $\triangle YXZ \cong \triangle WXZ$	5.
6. $\overline{YZ} \cong \overline{WZ}$	6.

Choose a reason from this list:

- Definition of angle bisector
- Definition of congruent triangles or CPCTC
- Given
- Given
- Reflexive property of congruence
- Side-Angle-Side congruence

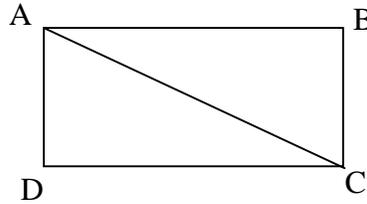
Different Methods of Proof

Two-Column Proofs (Continued)

2. Mark the given information on the diagram. Give a reason for each step in the two-column proof. Choose the reason for each statement from the list below.

Given: $\overline{AD} \cong \overline{BC}$
 $\overline{AB} \cong \overline{DC}$

Prove: $\overline{AD} \parallel \overline{BC}$



Statement	Reason
1. $\overline{AD} \cong \overline{BC}$	1.
2. $\overline{AB} \cong \overline{DC}$	2.
3. $\overline{AC} \cong \overline{AC}$	3.
4. $\triangle CAD \cong \triangle ACB$	4.
5. $\angle DAC \cong \angle BCA$	5.
6. $\overline{AD} \parallel \overline{BC}$	6.

Choose a reason from this list:

Definition of congruent triangles

Given

Given

If alternate interior angles are congruent then the lines are parallel.

Reflexive property of congruence

Side-Side-Side congruence

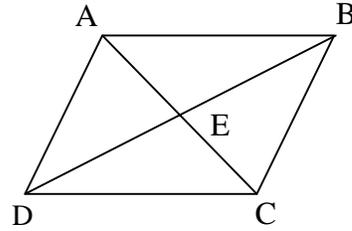
Different Methods of Proof

Two-Column Proofs (Continued)

3. Complete the following proof by filling in each statement. Remember to mark all given information on the diagram.

Given: ABCD is a parallelogram

Prove: $\triangle ABE \cong \triangle CDE$



Statement	Reason
1.	1. Given
2.	2. In a parallelogram, opposite sides are congruent.
3.	3. In a parallelogram, diagonals bisect each other.
4.	4. In a parallelogram, diagonals bisect each other.
5.	5. Side-Side-Side congruence

Choose a statement from this list:

$$\overline{AE} \cong \overline{EC}$$

$$ABCD \text{ is a parallelogram}$$

$$\overline{DE} \cong \overline{EB}$$

$$\triangle ABE \cong \triangle CDE$$

$$\overline{AB} \cong \overline{DC}$$

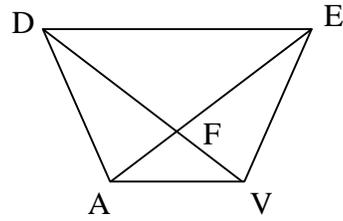
Different Methods of Proof

Two-Column Proofs (Continued)

4. Fill-in the statements and reasons for the following proof.

Given: $\overline{DE} \parallel \overline{AV}$
 $\triangle DAV \cong \triangle EVA$

Prove: DAVE is an isosceles trapezoid



Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

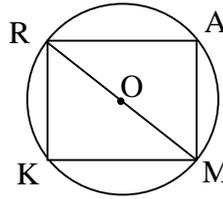
Possible Statements	Possible Reasons
DAVE is a trapezoid	Given
$\overline{DA} \cong \overline{EV}$	Definition of isosceles trapezoid
DAVE is an isosceles trapezoid	Given
$\triangle DAV \cong \triangle EVA$	Definition of trapezoid
$\overline{DE} \parallel \overline{AV}$	Definition of congruent triangles

Different Methods of Proof

Two-Column Proofs (Continued)

5. Complete the following proof.

Given: \overline{MR} is a diameter of $\odot O$
 $\overline{AR} \cong \overline{MK}$



Prove: $\triangle MAR \cong \triangle RKM$

Statement	Reason
1. \overline{MR} is a diameter of $\odot O$	1.
2. \widehat{MAR} and \widehat{MKR} are semicircles	2.
3. $\angle MAR$ and $\angle MKR$ are right angles	3.
4. $\angle MAR \cong \angle MKR$	4.
5. $\overline{MR} \cong \overline{MR}$	5.
6. $\overline{AR} \cong \overline{MK}$	6.
7. $\triangle MAR \cong \triangle RKM$	7.

Choose from this list of reasons.

- An angle inscribed in a semicircle is a right angle.
- All right angles are congruent
- Definition of a semicircle
- Given
- Given
- Hypotenuse-Leg Congruence
- Reflexive property of congruence

Different Methods of Proof

Lesson Plan – Practice with Proofs

Essential Questions

How can deductive reasoning be used to validate conjectures?

What methods can be used to organize a deductive argument?

Warm-Up/Opening Activity

Rewrite conjectures in if-then form.

Development of Ideas

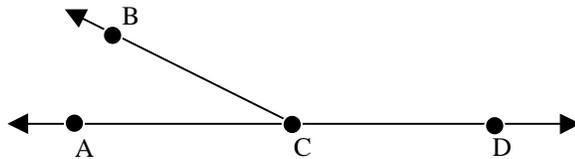
Investigate the relationship between an if-then statement and the given information and then prove information in a deductive proof.

Practice simple proofs.

Worksheet: Practice with Proofs

Answers: 1. a. If a linear pair includes one angle of 45° , then the other angle measures 135°

b.



c. Given: $\angle ACB = 45^\circ$

Prove: $\angle BCD = 135^\circ$

d. Given the fact that a linear pair is defined as two angles that add up to 180° . Since the two angles, $\angle ACB$ and $\angle BCD$ are a linear pair, they add up to 180° . Since we are told that $\angle ACB$ is 45° , then by subtraction we know that $\angle BCD$ must be 135° .

e. Statement

Reason

1. $\angle ACB$ and $\angle BCD$ are a linear pair 1. Given

2. $\angle ACB = 45^\circ$ 2. Given

3. $\angle ACB + \angle BCD = 180^\circ$ 3. Def. of linear pair

4. $45^\circ + \angle BCD = 180^\circ$ 4. Substitution prop.

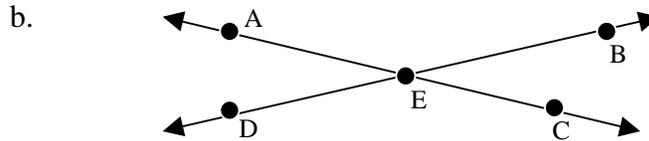
5. $\angle BCD = 135^\circ$ 5. Subtraction prop.

Different Methods of Proof

Development of Ideas (Continued)

Answers to Practice with Proof (Continued)

2. a. If two angles are supplements to the same angle, then the two angles are congruent.



- c. Given: $\angle DEA$ is supplementary to $\angle AEB$
 $\angle CEB$ is supplementary to $\angle AEB$

Prove: $\angle DEA \cong \angle CEB$

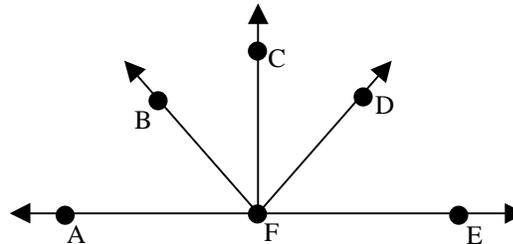
- d. Two angles being supplementary to the same angle means that $\angle DEA + \angle AEB = 180^\circ$ and that $\angle CEB + \angle AEB = 180^\circ$. By the transitive property, $\angle DEA + \angle AEB = \angle CEB + \angle AEB$. Since we know that $\angle AEB = \angle AEB$ by the reflexive property, then $\angle DEA = \angle CEB$ by the subtraction property and $\angle DEA \cong \angle CEB$ by the definition of angle congruence.

- e. Statements

1. $\angle DEA$ is supplementary to $\angle AEB$
2. $\angle CEB$ is supplementary to $\angle AEB$
3. $\angle DEA + \angle AEB = 180^\circ$
4. $\angle CEB + \angle AEB = 180^\circ$
5. $\angle DEA + \angle AEB = \angle CEB + \angle AEB$
6. $\angle DEA = \angle CEB$
7. $\angle DEA \cong \angle CEB$

3. a. If two angles are complements to congruent angles, then they themselves are congruent.

- b.



- c. Given: $\angle AFB$ is complementary to $\angle BFC$
 $\angle EFD$ is complementary to $\angle DFC$
 $\angle BFC \cong \angle DFC$

Prove: $\angle AFB \cong \angle EFD$

Different Methods of Proof

Development of Ideas (Continued)

Answers to Practice with Proof (Continued)

3. d. Since the two pair of given angles are complementary, then they each add up to 90° by the definition of complementary angles. By use of the transitive property, we can say that each pair of angle sums is equal to one another. Since one of each of the pair of angles are already congruent, then by the subtraction property, the other angle in each pair is also congruent.

e. Statements

1. $\angle BFC \cong \angle DFC$
2. $m\angle BFC = m\angle DFC$
3. $\angle AFB$ is complementary to $\angle BFC$
4. $\angle EFD$ is complementary to $\angle DFC$
5. $\angle AFB + \angle BFC = 90^\circ$
6. $\angle EFD + \angle DFC = 90^\circ$
7. $\angle AFB + \angle BFC = \angle EFD + \angle DFC$
8. $\angle AFB + \angle BFC = \angle EFD + \angle BFC$
9. $\angle AFB = \angle EFD$
10. $\angle AFB \cong \angle EFD$

Closure

What are the important elements in any proof?

Answer: Every proof must have givens and what is to be proved as well as logical reasoning to get from one to the other.

Compare and contrast flow-chart, paragraph, and two-column proofs.

Answer: All proofs use given statements and logical reasoning to prove statements. Flow-chart proofs allow multiple connects and pathways, paragraph proofs use sentences to demonstrate logical reasoning, and two-column proofs use a linear structure to go from given statements to what is to be proved.

Different Methods of Proof

Practice with Proof

1. Conditional statements are used often in geometry but are not always written in the if-then form that is needed for constructing a proof.

- a. For the conditional below, rewrite the statement in if-then form, and then label the hypothesis and the conclusion.

In a linear pair where one angle measures 45° , the other measures 135° .

- b. The second step in writing a proof is to draw and label a diagram that represents the given information. Draw and label a diagram for the statement. Remember to label the names of the angles, not just write their measures.

- c. The third step is to label the given and prove in terms of the diagram. Use the names of the angles in your statements.

Given:

Prove:

- d. Write a paragraph to explain to someone else why you know the conditional is true. Include the reason why you know each statement is true.

- e. Now complete the proof based on the explanation from part d.

Statement	Reason
1.	1. given
2.	2. given
3.	3.
4.	4.
5.	5.

Different Methods of Proof
Practice with Proof (Continued)

2. a. For the conditional, rewrite the statement in if-then form, and then label the hypothesis and the conclusion.

Supplements of the same angle are congruent.

- b. Draw and label a diagram for the statement. Remember to label the names of the angles.
- c. Write the given and prove in terms of the diagram. Use the names of the angles in your statements. Hint: There are two given statements.

Given:

Prove:

- d. Write a paragraph to explain to someone else why you know the conditional is true. Include the reason why you know each statement is true.

- e. Now complete the proof based on the explanation from part d.

Statements	Reasons
1.	1. Given
2.	2. Given
3.	3. Definition of supplementary
4.	4. Definition of supplementary
5.	5. Substitution property of equality
6.	6. Subtraction property of equality
7.	7. Definition of congruence

Different Methods of Proof

Practice with Proof (Continued)

3. a. For the conditional, rewrite the statement in if-then form, and then label the hypothesis and the conclusion.

The complements of congruent angles are congruent.

- b. Draw and label a diagram for the statement. Remember to label the names of the angles.
- c. Write the given and prove in terms of the diagram. Use the names of the angles in your statements. Hint: There are three given statements.

Given:

Prove:

- d. Write a paragraph to explain to someone else why you know the conditional is true. Include the reason why you know each statement is true.
- e. Now complete the proof based on the explanation from part d.

Statements	Reasons
1.	1. Given
2.	2. Definition of congruence
3.	3. Given
4.	4. Given
5.	5. Definition of complementary
6.	6. Definition of complementary
7.	7. Transitive property of equality
8.	8. Substitution property of equality
9.	9. Subtraction property of equality
10.	10. Definition of congruence

Different Methods of Proof

Lesson Plan – Coordinate Proofs

Essential Questions

How can deductive reasoning be used to validate conjectures?

What methods can be used to organize a deductive argument?

Warm-Up/Opening Activity

Use the Pythagorean Theorem to find the distance between A(5, 3) and B(-4, 1).

Development of Ideas

Recall the distance, slope, and midpoint formulas.

Place geometric figures on a coordinate plane so that calculations are simplified.

Justify geometric properties using coordinate proofs.

Worksheet: Introduction to Coordinate Proofs

- Answers:
- a. If a quadrilateral is a square, then the diagonals are congruent and are perpendicular bisectors of one another.
 - a. If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side of the triangle and has a length that is one-half of the length of the third side.

Worksheet: Coordinate Proofs

- Answers:
- Students should set up a parallelogram (with coordinates $(0, 0)$, $(a, 0)$, (b, c) , and $(a + b, c)$ for example), find the midpoints of two opposite sides $\left(\left(\frac{a}{2}, 0\right)\text{ and }\left(\frac{a+2b}{2}, c\right)\text{ in the example}\right)$ and then find slope of the two sides as well as the segment joining the midpoints to determine that they are all parallel.
 - Students should create a random quadrilateral, determine the midpoints of all four sides, find the slopes of the opposite sides, and find that opposite sides are parallel.

Different Methods of Proof

Development of Ideas

Answers to Coordinate Proofs (Continued)

3. Students should create a parallelogram, find the midpoint of each diagonal, and determine that it is the same point.
4. Students should create a rectangle and find the distance between opposite vertices, showing that the lengths of the diagonals are the same.
5. Students should create a random rhombus (being sure that all four lengths are the same), then determine the slope between the opposite vertices. The product of the slopes should be -1 to show that they are perpendicular.
6.
 - a. Students should create a trapezoid and find the midpoints of the non-parallel sides. Students should find the slope of the parallel sides as well as the slope of the segment connecting the midpoints to show that the three segments are parallel.
 - b. Using the distance formula, students should show that the length of the midsegment is one-half the length of the sum of the two parallel bases.

Closure

Describe the advantages and disadvantages of coordinate proofs.

Answer: The advantages of coordinate proofs is that they can be generalized to all possible contexts and they incorporate the use of algebraic reasoning. A disadvantage is that you have to be sure that you are selecting a diagram that does include all possible cases for the given statements.

Describe when and why you might want to double the coordinates of a figure when using a coordinate proof.

Answer: Sometimes doubling the coordinates will allow the use of fractions to be easier to use in the problem.

Different Methods of Proof

Introduction to Coordinate Proofs

Proofs involving midpoints, slope, and distance can be simplified by using analytic geometry. These proofs are called **coordinate proofs**. In a coordinate proof, the figure is drawn and labeled on a coordinate plane in a way that makes finding distances easy. Begin by placing one vertex of the figure at the origin. Place one side of the figure on the x -axis. Place parallel lines on either horizontal or vertical lines. Use a horizontal line and a vertical line for perpendicular lines. Once the figure has been placed on the coordinate plane, the distance formula can be used to measure distances, the midpoint formula can be used to locate points, and the slope formula can be used to determine parallel or perpendicular lines. Coordinate proofs rely on the premises of geometry plus the following properties from algebra.

Coordinate Geometry Formulas

The distance between points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

The midpoint of the segment joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

The slope m of a line through two points (x_1, y_1) and (x_2, y_2) , $x_1 \neq x_2$, is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

The slope of a horizontal line is zero.

The slope of a vertical line is undefined.

Two lines with slopes m_1 and m_2 are parallel if and only if $m_1 = m_2$.

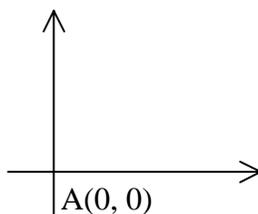
Any vertical line is perpendicular to any horizontal line.

Two non-vertical lines are perpendicular if and only if their slopes are negative reciprocals of each other.

1. Write a coordinate proof of the conjecture:

The diagonals of a square are congruent and are perpendicular bisectors of each other.

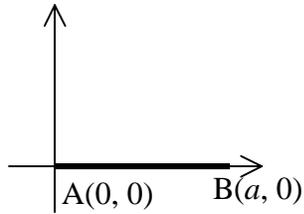
- a. For the conditional, rewrite the statement in if-then form, and then label the hypothesis and the conclusion.
- b. Place and label the figure on the coordinate plane.
 - (1) Place one vertex, point A, at the origin.



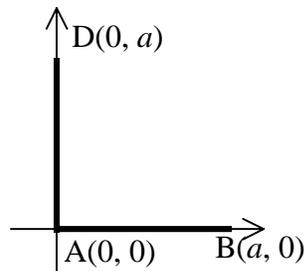
Different Methods of Proof

Introduction to Coordinate Proofs (Continued)

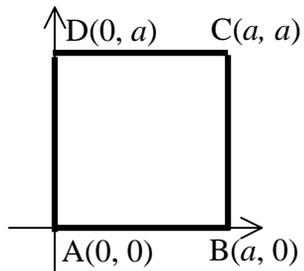
- (2) Place a second vertex, point B, on the x-axis. This simplifies calculations because the y-coordinate of this point is 0.



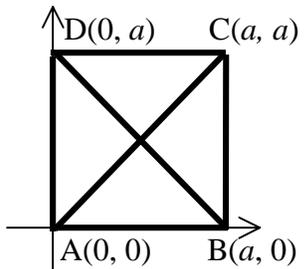
- (3) \overline{AD} and \overline{AB} must be perpendicular. Since \overline{AB} lies on the x-axis, \overline{AD} must lie on the y-axis a units above point A.



- (4) Place point C a units above point B.



- (5) Draw the diagonals.



Different Methods of Proof

Introduction to Coordinate Proofs (Continued)

- b. Write the given and prove of the conditional statement in terms of the diagram.

Given: Square ABCD with diagonals \overline{AC} and \overline{BD}

Prove: $\overline{AC} \cong \overline{BD}$

$\overline{AC} \perp \overline{BD}$

\overline{AC} and \overline{BD} bisect each other

- c. Use the distance formula to find the lengths of the two diagonals.

$$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(a - 0)^2 + (a - 0)^2} = \sqrt{2a^2} = a\sqrt{2}$$

$$BD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(a - 0)^2 + (0 - a)^2} = \sqrt{2a^2} = a\sqrt{2}$$

So, by the definition of congruence, $\overline{AC} \cong \overline{BD}$ because they both have the same lengths.

- c. Use the midpoint formula to find the midpoints of the two diagonals.

$$\text{Midpoint of } \overline{AC} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{0 + a}{2}, \frac{0 + a}{2} \right) = \left(\frac{a}{2}, \frac{a}{2} \right)$$

$$\text{Midpoint of } \overline{BD} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{0 + a}{2}, \frac{a + 0}{2} \right) = \left(\frac{a}{2}, \frac{a}{2} \right)$$

So, \overline{AC} and \overline{BD} bisect each other because both segments have the same midpoint.

- d. Use the slope formula to compare the slopes of the two diagonals.

$$\text{Slope of } \overline{AC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{a - 0}{a - 0} = 1$$

$$\text{Slope of } \overline{BD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{a - 0}{0 - a} = -1$$

So, $\overline{AC} \perp \overline{BD}$ because the product of the slopes of the two segments is -1 .

Therefore, the diagonals of a square are congruent and are perpendicular bisectors of each other.

2. Write a coordinate proof of the conditional statement:

The segment that joins the midpoints of two sides of a triangle
(1) is parallel to the third side of the triangle, and
(2) has a length equal to half the length of the third side.

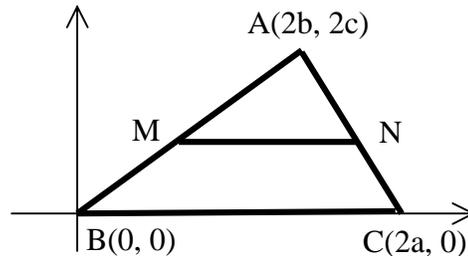
- a. For the conditional, rewrite the statement in if-then form, and then label the hypothesis and the conclusion.

Different Methods of Proof

Introduction to Coordinate Proofs (Continued)

- b. Draw and label a figure on the coordinate plane.

Hint: The algebra to calculate the coordinates of the midpoints of the two sides of the triangle can be simplified if you multiply each of the coordinates of the vertices of the triangle by two.



- c. Write the given and prove in terms of the diagram.

Given: Triangle ABC
M is the midpoint of \overline{AB}
N is the midpoint of \overline{AC}

Prove: (1) $\overline{MN} \parallel \overline{BC}$

$$(2) MN = \frac{BC}{2}$$

- d. Use the midpoint formula to find the coordinates of the midpoints of the two congruent sides of the triangle.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{2b + 0}{2}, \frac{2c + 0}{2} \right) = (b, c)$$

$$N = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{2a + 2b}{2}, \frac{2c + 0}{2} \right) = (a + b, c)$$

- e. Use the slope formula to compare the slopes of the two segments.

$$\text{Slope of } \overline{MN} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{c - c}{a + b - b} = \frac{0}{a} = 0$$

$$\text{Slope of } \overline{BC} = \frac{0 - 0}{2a - 0} = \frac{0}{a} = 0$$

So, $\overline{MN} \parallel \overline{BC}$ because the two segments have equal slopes

Different Methods of Proof
Introduction to Coordinate Proofs (Continued)

- f. Use the distance formula to find the lengths of the two segments.

$$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(a + b - b)^2 + (0 - 0)^2} = \sqrt{(a)^2 + (0)^2} = a$$

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2a - 0)^2 + (0 - 0)^2} = \sqrt{(2a)^2 + (0)^2} = 2a$$

$$\text{So, } MN = \frac{BC}{2} \text{ because } a = \frac{1}{2}(2a).$$

Therefore, the segment that joins the midpoints of two sides of a triangle is (1) parallel to the third side of the triangle, and (2) equal in length to one-half the length of the third side.

Different Methods of Proof

Coordinate Proofs

Use coordinate proofs to justify the following conditional statements.

1. The segment joining the midpoints of two opposite sides of a parallelogram is parallel to the other two sides.
2. The quadrilateral formed by joining, in order, the midpoints of the sides of a quadrilateral is a parallelogram.
3. The diagonals of a parallelogram bisect each other.
4. The diagonals of a rectangle are congruent.
5. The diagonals of a rhombus are perpendicular.
6. The mid-segment of a trapezoid is
 - a. parallel to each of the bases of the trapezoid, and
 - b. equal in length to one-half the sum of the lengths of the two bases.