Different Methods of Construction

Objectives

For the students to be able to perform geometric constructions using a variety of construction tools, investigate the properties of perpendicular bisectors and angle bisectors, investigate points of concurrency in triangles, and apply geometric constructions to real-life situations.

Core Learning Goals

2.1.4 The student will construct and/or draw and/or validate properties of geometric figures using appropriate tools and technology.

2.2.3 The student will use inductive or deductive reasoning.

Materials Needed

Worksheets, compass, straightedge, patty paper, Mira™

Pre-requisite Concepts Needed

Basic vocabulary: line, line segment, angle, ray, distance, congruence, perpendicular, bisector, midpoint, distance between two points, distance between a point and a line, median of a triangle, altitude of a triangle

Students should understand the relationship of a figure and its image after a reflection over a line.

Approximate Time

Two to three 50-minute lessons or one to two 90-minute lessons

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Lesson Plan – Different Methods of Construction

Essential Questions

What are the measuring tools used in geometry?

How and when are these measuring tools used in geometry?

What are the construction tools used in geometry?

How and when are these construction tools used in geometry?

Warm-Up/Opening Activity

Discuss how you would find the distance between two students in the classroom. Students should understand that distance is always measured as the shortest distance. The shortest distance between two points is the straight-line distance.

Discuss how you measure the distance from a student in the classroom to a wall of the classroom. Students should understand that the shortest distance between a point and a line (or a plane) is the perpendicular distance from the point to the line (or plane).

Development of Ideas

Discuss the appropriate uses of geometric construction tools.

A straightedge is only used for drawing line segments, not for measuring.

A compass can be used for drawing a circle with a given center and a given radius measured from that center or to copy a line segment or transfer a distance from one place to another.

Patty paper and Miras™ can be used to transfer distances or replicate congruent figures. They cannot be used for drawing circles.

Replicate a segment using the different geometric construction tools.
Worksheet: **Review of Basic Constructions**

1. Steps for patty paper construction
   (1) Trace $\overline{AB}$ onto a piece if patty paper. Label it $\overline{A'B'}$.
   (2) Make heavy dots at A′ and B′.
   (3) Turn the patty paper over so that $\overline{A'B'}$ is on the underside of the patty paper. Place the patty paper so that $\overline{A'B'}$ is close to $\overline{AB}$.
   (4) Transfer points A′ and B′ to the original paper by rubbing the patty paper at points A′ and B′.
   (5) Connect the images of A′ and B′ with a straightedge. Label this new segment $\overline{A''B''}$.

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Development of Ideas (Continued)

2. Steps for Mira construction
   (1) Place the beveled edge of the Mira™ parallel to and slightly above $\overline{CD}$.
   (2) Look through the Mira™ and locate the reflection of $\overline{CD}$. Draw the reflection of $\overline{CD}$ with a straightedge. Label it $\overline{C'D'}$.

3. Steps for compass and straightedge construction
   (1) Draw $\overrightarrow{E'X}$ that is longer than $\overline{EF}$
   (2) Set compass to the length of $\overline{EF}$
   (1) Set point of the compass on E′
   (4) Mark the distance of $\overline{EF}$ on $\overrightarrow{E'X}$
   (5) Label the intersection as F′

4. Answers will vary. Examine student reasons for "Why?"

Determine right angles and perpendicular lines using patty paper.

Worksheet: **Patty Paper Investigations, 1-2**

Answers:
1. The corner of patty paper is a right angle.
2. Corners can be matched to see if the angle is a right angle.

Determine distance using patty paper.

Worksheet: **Patty Paper Investigations, 3-6**

Answers:
3. Linear, perpendicular, perpendicular
4. Place point A on one edge of the patty paper. Line up the edge of the patty paper that is perpendicular to the edge along A with line *m*. Mark point A on the patty paper. Using the same two edges of the patty paper, repeat the process for point B. Points A and B are now marked on the same edge of the patty paper. Since point A is closer to the corner used to line up with line *m*, point A is a shorter distance from line *m* than point B.
5. Repeat the process to measure the distance of point B from line *m*. Slide the patty paper along line *m* and make point C using the copy of point B.
6. No, I copied and labeled points A and B on the patty paper. Keeping the patty paper on point A, I rotated the patty paper to try to line up point B on the patty paper with point C. It was impossible to do since AB < AC.

Determine whether figures are congruent using patty paper.

Worksheet: **Patty Paper Investigations, 7**

Answer:
7. Figure 1 and 3 are congruent. You could measure sides and angles using patty paper to demonstrate any of the following: ASA, SAS, SSS, or AAS congruence.

Different Methods of Construction

Development of Ideas (Continued)

Create congruent figures using patty paper.

Worksheet: **Patty Paper Investigations, 8**

Answer: 8. You could use the patty paper to measure sides and/or angles to demonstrate any of the following triangle congruence theorems: SAS, ASA, SSS, AAS.

Construct perpendicular lines and angle bisectors using patty paper.

Worksheet: **Constructions Using Patty Paper**

Answers: Definitions – right angles, congruent, congruent

Construct a line perpendicular to a given line

Step 4: Perpendicular. Perpendicular lines can be demonstrated using patty paper by matching the corner of the paper to demonstrate right angles.

Construct a line through a point perpendicular to a given line

Step 4: Perpendicular. Perpendicular lines can be demonstrated using patty paper by matching the corner of the paper to demonstrate right angles.

Construct a perpendicular bisector of a given line segment

Step 4: Perpendicular

Step 5: Congruent segments

Step 6: Midpoint

Step 8: Congruent segments. This can be shown using patty paper by copying the length of one segment and overlaying it over the second segment and confirming congruence.

Step 10: Any point on the perpendicular bisector of a segment is equidistant from the endpoint of the segment.

Construct the bisector of an angle

Step 4: Congruent angles. This can be shown by tracing one angle and then overlaying it over the second angle to confirm congruence.

Step 5: Point T is equidistant to each side of the angle. This can be shown using patty paper by placing point T on one edge of the patty paper. Line up the edge of the patty paper that is perpendicular to the edge along T with one side of the angle. Mark point T on the patty paper. Repeat the process with the other side of the angle to confirm equal distance.

Step 6: Equidistant

Step 7: Every point on the bisector of an angle is equidistant from both sides of the angle.

Different Methods of Construction

Development of Ideas (Continued)

Construct perpendicular lines and angle bisectors using a compass and straight edge.

Worksheet: **Constructions Using Compass and Straight Edge**

Answers: Construct a line through a point perpendicular to a given line

Step 4: Perpendicular. Perpendicular lines can be justified by measuring the right angles.

Constructing the perpendicular bisector of a line segment

Step 5: Perpendicular. The perpendicular bisector can be justified by measuring the right angles and congruent segments.

Construct an angle bisector

Step 4: Congruent angles. Angle congruence can be justified by measurement.

Construct perpendicular lines and angle bisectors using a Mira.

Worksheet: **Constructions Using Miras**

Answers: Construct a line through a point perpendicular to a given line

Step 5: Perpendicular. Perpendicular lines can be justified by measuring the right angles.

Constructing the perpendicular bisector of a line segment

Step 4: Perpendicular. The perpendicular bisector can be justified by measuring the right angles and congruent segments.

Construct an angle bisector

Step 5: Congruent angles. Angle congruence can be justified by measurement

Worksheet: **Constructions**

Note: This activity could be completed as a jigsaw activity. Students could complete one or two of the problems, assigned by the teacher. The students could then share their solutions with the class in the next lesson.

Possible Answers/Samples:

1. I constructed the perpendicular bisector because the perpendicular bisector of $\overline{AC}$ is the set of points that are equidistant from A and C.
2. I constructed the angle bisector because the angle bisector is the set of points equidistant from the two rays of an angle.
3. I constructed the perpendicular bisectors of sides $\overline{AB}$ and $\overline{BC}$ to find their midpoints. I connected the midpoints to determine the midsegment.
4. First, construct a copy of $\overline{AB}$. Then, at A, construct a perpendicular line. Then copy the length of $\overline{AC}$ onto the perpendicular. Next, connect the points B and C together.
5. First, construct a copy of $\overline{AB}$. Then, at A, construct a perpendicular line. Then, at B, copy the length of $\overline{AC}$ until it intersects the perpendicular. This intersection is the point C.

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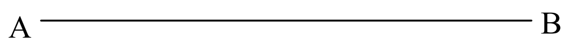
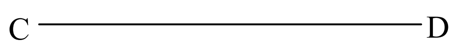
Review of Basic Constructions

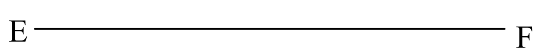
1. Construct a segment congruent to $\overline{AB}$ using patty paper. Use mathematics to explain how you constructed the line segment. Use words, symbols, or both in your explanation.



2. Construct a segment congruent to $\overline{CD}$ using a Mira™. Use mathematics to explain how you constructed the line segment. Use words, symbols, or both in your explanation.



3. Construct a segment congruent to $\overline{EF}$ using a compass and straightedge. Use mathematics to explain how you constructed the line segment. Use words, symbols, or both in your explanation.



4. Which of these methods do you prefer to use to construct a segment congruent to a given segment? Why?

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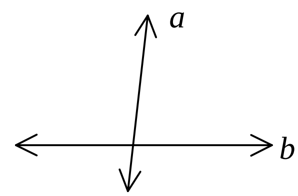
Patty Paper Investigations

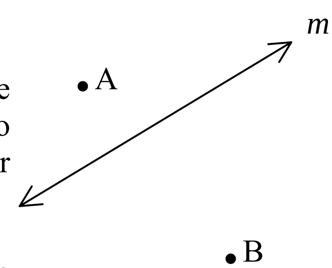
1. What is the shape of a corner of a sheet of patty paper?

2. Explain how you could use your patty paper to determine whether line *a* is perpendicular to line *b*.



3. When you measure the distance between two figures, you always measure the shortest distance. The shortest distance between two points is the ___________________ distance. The shortest distance between a point and a line is the ___________________ distance between that point and the line. The shortest distance between a point and a plane is the _____________________ distance between that point and the plane.

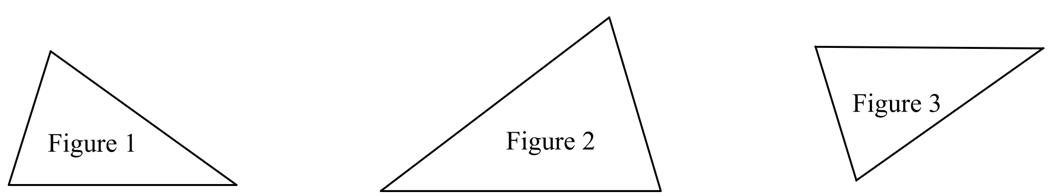
4. Explain how you could use patty paper to determine which point, A or B, lies the shortest distance from line *m*.



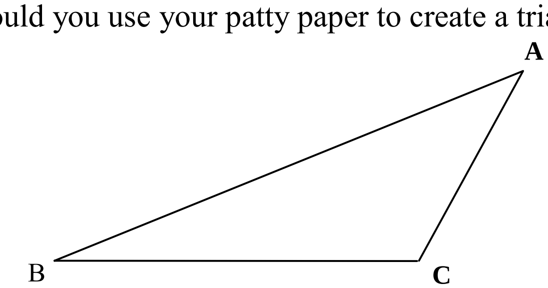
5. Use only your patty paper to locate a point, C, that is the same distance from line *m* as point B. Use mathematics to explain how you located point C. Use words, symbols, or both in your explanation.

6. Is the distance from point C to point A the same as the distance from point C to point B? Use your patty paper to justify your answer.

7. Two figures are congruent if all pairs of corresponding parts are congruent. Are any of the triangles shown below congruent? How could you use your patty paper to justify your answer?



8. Explain how could you use your patty paper to create a triangle congruent to triangle ABC?



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Constructions Using Patty Paper

Complete the following definitions:

If two lines are perpendicular, then any two adjacent angles formed are _____________.

If a line segment is bisected, then the two segments formed are ____________________.

If an angle is bisected, then the two adjacent angles formed are _________________.

Construct a line perpendicular to a given line

Step 1: Draw a line on your patty paper. Label the line *m*.

Step 2: Fold your patty paper so that the two parts of line *m* lie exactly on top of each other. Crease the patty paper on the fold.

Step 3: Open the patty paper and draw a line on the crease. Label this line *n*.

Step 4: What is the relationship of line *n* to line *m*? Describe how you could use the corner of your patty paper to justify this relationship.

Construct a line through a given point perpendicular to a given line

Step 1: Draw a line on your patty paper. Label the line *m*. Draw a point on the patty paper that is **not** on line *m* and label this point P.

Step 2: Fold your patty paper so that the two parts of line *m* lie exactly on top of each other. Slide the patty paper so that point P will be on the fold. Crease the patty paper on the fold.

Step 3: Open the patty paper and draw a line on the crease. Label this line *n*.

Step 4: What is the relationship of line *n* to line *m*? Describe how you could use your patty paper to justify this relationship.

Construct a perpendicular bisector of a given line segment using patty paper

Step 1: Draw a line segment on your patty paper. Label the line segment AB.

Step 2: Fold the patty paper so that points A and B, the two end points of the segment you drew on the patty paper, coincide with each other. Crease the paper along the fold.

Step 3: Open the patty paper and draw a line on the crease. Label this line *k*. Label the intersection of line *k* with line segment AB as point M.

Different Methods of Construction

Constructions Using Patty Paper (Continued)

Construct a perpendicular bisector of a given line segment using patty paper continued

Step 4: What is the relationship of line *k* to $\overline{AB}$?

Step 5: What is the relationship between $\overline{AM}$ and $\overline{BM}$?

Step 6: Point M is the ________________ of $\overline{AB}$.

Step 7: Select a point on line *k*. Label this point X.

Step 8: What is the relationship between AX and BX? Describe how you could use your patty paper to justify this relationship.

Step 9: Select a different point on line *k* and repeat steps 7 and 8.

Step 10: Write a conjecture stating the relationship between any point on the perpendicular bisector of a line segment and the endpoints of that line segment.

Construct the bisector of an angle using patty paper

Step 1: Draw an angle on a sheet of patty paper. Label this angle $\angle QRS$.

Step 2: Fold your patty paper so that the two sides of the angle, $\overrightarrow{RQ}$ and $\overrightarrow{RS}$, coincide. Crease the paper along the fold.

Step 3: Unfold your patty paper. Select a point on the interior of $\angle QRS$ that lies on the crease. Label this point T. Draw ray RT.

Step 4: What is the relationship between $\angle QRT$ and $\angle SRT$? How can you use your patty paper to justify this relationship?

Step 5: What is the relationship between the distances from point T to each of the sides of the angle? Using your patty paper, explain how you determined this relationship.

Step 6: Select another point on ray RT. Label this point W. What is the relationship between the distances from point W to each of the sides of the angle?

Step 7: Write a conjecture comparing the distances from a point that lies on an angle bisector to each of the sides of the angle.

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Constructions Using Compass and Straight Edge

Construct a line through a given point perpendicular to a given line

Step 1: Draw a line on your paper. Label the line *m*. Draw a point on your paper that is **not** on line *m* and label this point P.

Step 2: Open your compass to a measure greater than the distance from point P to line m. Place the point of the compass on point P. Draw an arc that intersects line *m* at two points. Label these points A and B.

Step 3: Place the compass point at A and draw an arc on the opposite side of the line from point P.

Step 4: Leave the arc compass opening the same. Move the point of the compass to point B. Draw an arc that intersects the arc you made in step 3. Label the point of intersection of the two arcs as point X.

Step 5: Draw line PX.

Step 6: What is the relationship of line PX to line *m*? Use mathematics to justify your answer.

Construct the perpendicular bisector of a line segment

Step 1: Draw a line segment on your paper. Label the endpoints A and B.

Step 2: Open your compass to a measure greater than half the distance between A and B. Place the tip of your compass at A and draw a large arc that intersects $\overline{AB}$.

Step 3: Leave the compass opening the same. Move the point of the compass to point B. Draw an arc that intersects the arc you made in step 2 at two points. Label the points of intersection of the two arcs as points X and Y.

Step 4: Draw line XY.

Step 5: What is the relationship of line XY to $\overline{AB}$? Use mathematics to justify your answer.

Construct an angle bisector

Step 1: Draw an angle on your paper. Label the vertex of the angle point A.

Step 2: Place your compass point at A and draw an arc that passes through both sides of the angle. Label the intersection points of the arc with the sides of the angle as B and C.

Step 3: Place your compass point first at B and then at C. Using a radius greater than half the distance from B to C, draw arcs that intersect in the interior of $\angle A$. Label the intersection of the two arcs as point F. Draw ray AF.

Step 4: What is the relationship between $\angle CAF$ and $\angle BAF$? Use mathematics to justify your answer.

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Constructions Using Miras™

Construct a line through a given point perpendicular to a given line

Step 1: Draw a line on your paper. Label the line *m*. Draw a point on your paper that is **not** on line *m* and label this point P.

Step 2: Place a Mira™ on your paper so that the beveled edge of the Mira™ coincides with point P.

Step 3: Slide the Mira™ so that point P remains on the beveled edge of the Mira™ and all the points of the line *m* coincide with the reflection of line *m* in the Mira™.

Step 4: Using the beveled edge of the Mira™ as a guide, draw the line of reflection. Label this line *n*.

Step 5: What is the relationship of line *n* to $\overline{AB}$? Use mathematics to justify your answer.

Construct the perpendicular bisector of a line segment

Step 1: Draw a line segment on your paper. Label the endpoints A and B.

Step 2: Place a Mira™ on your paper so that the reflection of point A coincides with point B in the Mira™.

Step 3: Using the beveled edge of the Mira™ as a guide, draw the line of reflection. Label this line *n*.

Step 4: What is the relationship of line *n* to line *m*? Use mathematics to justify your answer.

Construct an angle bisector

Step 1: Draw an angle on your paper. Label the angle $\angle ABC$.

Step 2: Place the Mira™ on your paper so that the vertex of the angle coincides with the beveled edge of the Mira™.

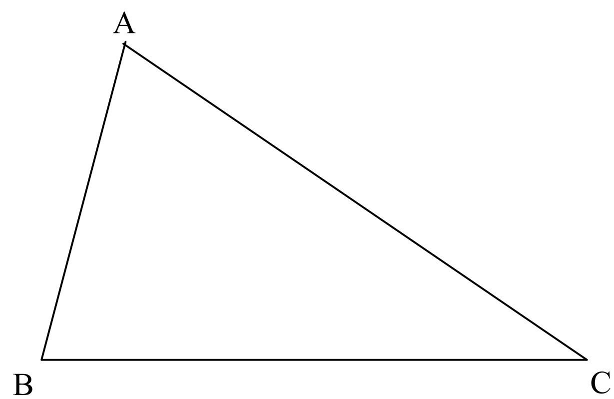
Step 3: Slide the Mira™ so that ray BA coincides with ray BC.

Step 4: Using the beveled edge of the Mira™ as a guide, draw the ray that has B as its endpoint and that passes through the interior of $\angle ABC$. Label the ray BF.

Step 5: What is the relationship between $\angle CBF$ and $\angle ABF$? Use mathematics to justify your answer.

Different Methods of Construction

Constructions



1. Construct all the points equidistant from points A and C. Use mathematics to justify your construction.

2. Construct all the points equidistant from ray BA and ray BC. Use mathematics to justify your construction.

3. Construct a mid-segment of triangle ABC that is parallel to $\overline{AC}$. Explain the steps you used in your construction. Use words, symbols, or both in your explanation.

4. Construct a right triangle that has legs of lengths AB and AC. Explain the steps you used in your construction. Use words, symbols, or both in your explanation.

5. Construct a right triangle that has a leg of length AB and a hypotenuse of length AC. Explain the steps you used in your construction. Use words, symbols, or both in your explanation.

Lesson Plan – Concurrent Lines in Triangles

Essential Questions

How can the properties of angle bisectors, perpendicular bisectors, and concurrent lines be applied to real-world situations?

Warm-Up/Opening activity

Review or define the following terms:

- Median of a triangle
- Altitude of a triangle
- Concurrent lines

Materials Needed

Heavy paper to create sturdy triangles that can be balanced without collapsing. Manila folders work well. Cut each folder along the crease. Give one side of a folder to each student. Encourage students to draw large triangles on their paper.

Development of Ideas

Construct and investigate the properties of concurrent lines in a triangle.

Divide the students into groups of 3. Assign each student in the group only one type of triangle (acute, right, or obtuse) to investigate.

Worksheet: **Concurrent Lines in Triangles**

Answers:

- Centroid
  - The median of a triangle is a segment that has as its endpoints a vertex of the triangle and the midpoint of the opposite side.
  - The medians of a triangle meet inside a triangle.
  - The lengths of the pieces of the median are in a 2:1 ratio – the point of concurrence is 2/3 of the distance from the vertex to the midpoint of the opposite side.
  - The concurrence point of the medians is the center of gravity of the triangle.
- Circumcenter
  - The point of concurrence of the perpendicular bisectors does not always meet inside the circle.

Development of Ideas (Continued)

Answers to Concurrent Lines in Triangles (Continued)

Circumcenter (Continued)

The point of concurrence of the perpendicular bisectors is equidistant from each vertex of the triangle.

Yes, the circle, using point of concurrence of the perpendicular bisectors as its center, circumscribes the triangle.

It is called the circumcenter because it is the center of the circle that circumscribes the triangle, where each of the vertices of the triangle is on the circle.

The circumcenter of a triangle is equidistant from the three vertices of a triangle.

Incenter

The point of concurrence of the angle bisectors falls inside of the triangle and is equidistant from the sides (perpendicular distance) of the triangle.

The circle that is drawn using the incenter is tangent to the three sides of the triangle because it touches each side at exactly one point. Therefore, it is inscribed in the triangle.

The point of concurrence of the angle bisectors is the center of the circle that is **in**side the triangle.

The incenter of a triangle is equidistant form the three sides of a triangle.

Orthocenter

The point of concurrence of the altitudes of a triangle can be inside, outside, or on the triangle.

Other words with the ortho- prefix inculde orthopedic, orthodontic, orthodox, orthogonal, orthoscopic.

Ortho- means straight, regular, or upright.

Summarize the properties of concurrent lines in acute, right, and obtuse triangles.

Worksheet: **Concurrent Lines in Triangles Chart**

Answers: The point of concurrency of the medians of a triangle meet inside the triangle at the point called the centroid, or the center of gravity.

The point of concurrency of the perpendicular bisectors meet inside, outside, or on the triangle at the point called the circumcenter, which is the center of the circumscribed circle about the triangle.

Different Methods of Construction

Development of Ideas (Continued)

Answers to Concurrent Lines in Triangles Chart (Continued)

The point of concurrency of the angle bisectors of a triangle meet inside the triangle at the incenter, which is the center of the circle inscribed in the triangle (meets all three sides of the triangle).

The point of concurrency of the altitudes of a triangle meet either inside, outside, or on the triangle and is called the orthocenter of the triangle.

Supplemental Activities/Resources

Investigate properties of the orthocenter using technology

Worksheet: **The Nine-Point Circle**

Worksheet: **Orthocenter Investigation**

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Concurrent Lines in Triangles

Centroid

Step 1: A *median of a triangle* is _____________________.

Step 2: Draw a triangle on your paper as directed by your teacher.

Step 3: Construct the three medians of the triangle.

Step 4: *Concurrent lines* are three or more coplanar lines that intersect in the same point. Are the three medians of the triangle concurrent? If so, is the point of concurrency inside, outside, or on the triangle?

Step 5: The point of concurrency of the medians of a triangle is called the *centroid* of the triangle. The centroid divides each median into two segments. Use your patty paper or a compass to compare the lengths of these two segments on each of the medians. What is the relationship between the lengths of these two segments? What is the relationship between each of these segments and the median?

Step 6: Write a conjecture stating the relationship between the two segments of each median formed by the centroid of the triangle.

Step 7: Draw a large triangle on a piece of heavy paper provided by your teacher. Use scissors to cut out your triangle.

Step 8: Balance your triangle on the tip of your pencil. The point at which the triangle is balanced is called the *center of gravity* of the triangle. Label the center of gravity of your triangle as point G.

Step 9: Use a ruler to locate the midpoint of each side of the triangle. Draw the three medians of the triangle.

Step 10: What is the relationship between the center of gravity of your triangle and the intersection of the three medians of the triangle? Compare your results to the other members of your group.

Step 11: State a second conjecture about the point of concurrency of the medians of a triangle.

Circumcenter

Step 1: Draw a triangle on your paper as directed by your teacher.

Step 2: Construct the perpendicular bisector of each of the sides of the triangle.

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Concurrent Lines in Triangles (Continued)

Circumcenter (Continued)

Step 3: Are the three perpendicular bisectors of the sides of the triangle concurrent? If so, is the point of concurrency inside, outside, or on the triangle?

Step 4: What is the relationship between the distances between the point of concurrency of the perpendicular bisectors of the sides of the triangle and each of the vertices of the triangle?

Step 5: Use your compass to construct a circle. Use the point of concurrency as the center of the circle. Set the radius of the compass so that it passes through one of the vertices of the triangle.

Step 6: Does your circle pass through the other two vertices of the triangle?

Step 7: A circle is circumscribed about a triangle if and only if it passes through each of the vertices of the triangle. Does the circle you drew in step 5 circumscribe your triangle? Use mathematics to justify your answer,

Step 8: The point of concurrency of the perpendicular bisectors of the sides of a triangle is called the circumcenter of the triangle. Why do think this name was selected as the point of concurrency?

Step 9: The circumcenter of a triangle is equidistant from the three _____________ of a triangle.

Incenter

Step 1: Draw a triangle on your paper as directed by your teacher.

Step 2. Construct the three angle bisectors of the triangle.

Step 3: Are the three angle bisectors of the triangle concurrent? If so, is the point of concurrency inside, outside, or on the triangle?

Step 4: What is the relationship between the distances between the point of concurrency of the angle bisectors of the triangle and each of the sides of the triangle? (Remember: The distance between a point and a line segment is always the _________________ distance.)

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Concurrent Lines in Triangles (Continued)

Incenter (Continued)

Step 5: Use your compass to construct a circle. Use the point of concurrency as the center of the circle. Use the distance from the point of concurrency of the angle bisectors to a side of the triangle as the radius. Therefore, you must first construct a segment from the point of concurrency perpendicular to a side of the triangle.

Step 6: Is your circle tangent to each of the sides of the triangle? Use mathematics to justify your answer.

Step 7: A circle is inscribed within a polygon if and only if it touches each side of the polygon at exactly one point. Is the circle you drew in step 5 inscribed in your triangle?

Step 8: The point of concurrency of the angle bisectors of a triangle is called the *incenter* of the triangle. Why do think this name was selected as the point of concurrency?

Step 9: The incenter of a triangle is equidistant from the three _____________ of a triangle.

Orthocenter

Step 1: Define *altitude* of a triangle.

Step 2: Draw a triangle on your paper as directed by your teacher.

Step 2. Construct the three altitudes of the triangle.

Step 3: Are the lines that contain the three altitudes of the triangle concurrent? If so, is the point of concurrency inside, outside, or on the triangle?

Step 4: The point of concurrency of the three altitudes of a triangle is called the orthocenter. Can you think of any other words that begin with the prefix *ortho*? What do you think the prefix *ortho* means?

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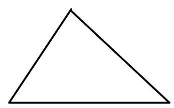
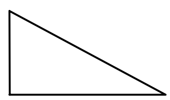
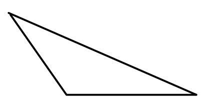
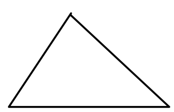
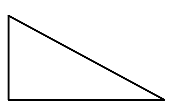
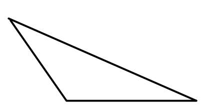
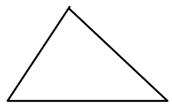
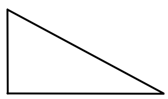
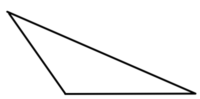
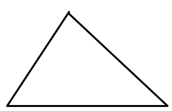
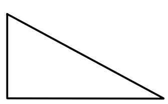
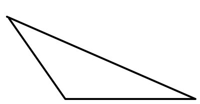
Concurrent Lines in Triangles Chart

Complete the table below for concurrent lines in triangles. Sketch the three indicated lines on the acute, right and obtuse triangles shown. Describe the location of the point of concurrency in each of these triangles. List the special features of each of the points of concurrency.

Lines	Drawing	Location of Point of Concurrency	Name of Point of Concurrency	Special Features of Point of Concurrency
Medians				
				
				
Perpendicular Bisectors				
				
				

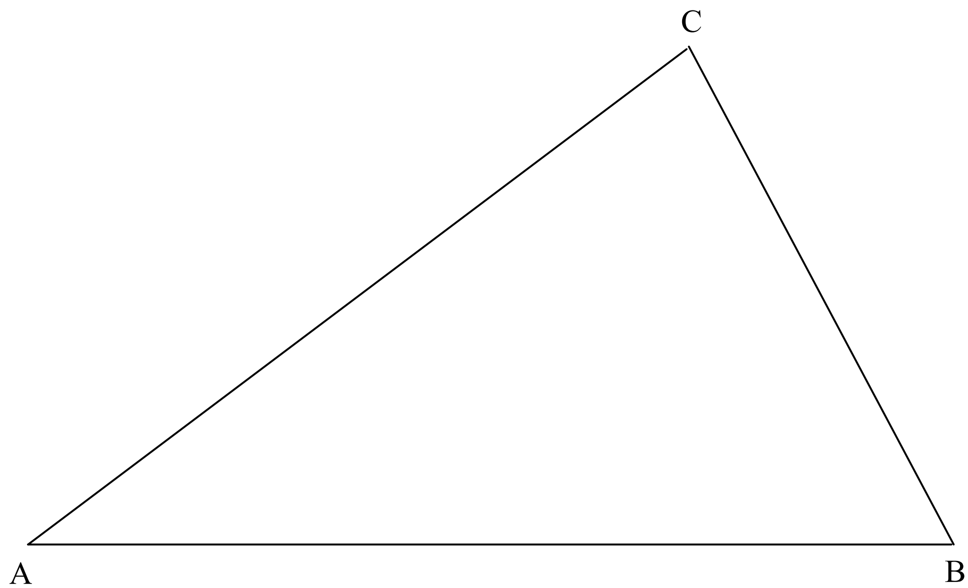
Different Methods of Construction

Concurrent Lines in Triangles Chart (Continued)

Lines	Drawing	Location of Point of Concurrency	Name of Point of Concurrency	Special Features of Point of Concurrency
Angle Bisectors				
				
				
Altitudes				
				
				

Different Methods of Construction

The Nine-Point Circle



1. Construct the perpendicular bisectors of the sides of ΔABC. Let X be the midpoint of $\overline{AB}$, Y the midpoint of $\overline{BC}$, and Z the midpoint of $\overline{AC}$. Let Q be the circumcenter of ΔABC.

2. Construct the three altitudes of ΔABC. Let K be the point where the altitude from C intersects $\overline{AB}$, let L be the point where the altitude from A intersects $\overline{BC}$, and let M be the point where the altitude from B intersects $\overline{AC}$. Let P be the orthocenter of ΔABC.

2. Construct the midpoints of $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$. Let D be the midpoint of $\overline{AP}$, E be the midpoint of $\overline{BP}$, and F the midpoint of $\overline{CP}$.

4. Points X, K, E, L, Y, F, M, Z, and D should lie on a circle. To find the center of the circle, draw $\overline{PQ}$ and construct the midpoint of $\overline{PQ}$. Call the midpoint O.

5. With O as center and OX as radius, draw the *Nine-Point Circle* through the points listed in Step 4.

Different Methods of Construction

Orthocenter Investigation

Step 1: Use a geometry software package to draw a triangle.

Step 2: Construct the three altitudes of the triangle. Construct the point of concurrency of these three altitudes. Label this point A. Point A is the _____________ of the triangle.

Step 3: Hide the three altitudes of the triangle.

Step 4: Draw a segment from each vertex of the triangle to the orthocenter, forming three new triangles within the original triangle.

Step 5: Construct the orthocenter of one of the new triangles formed in step 4. Where is this orthocenter?

Step 6: Repeat step 5 for the other two triangles formed in step 3. Where is the orthocenter for each of these triangles?

Step 7: Drag the vertices of the original triangle. What is the relationship between the four orthocenters?

Step 8: The orthocenter of any triangle formed by two vertices of another triangle and the orthocenter of that triangle is_______________.

Step 9: Conjecture: If one point in a given set of four points is the orthocenter of the other three, then _______________.