Constructions, Sketches, and Drawings

Objective

For the students to understand the difference between constructions, sketches, and drawings.

Core Learning Goals

2.1.4 The student will construct and/or draw and/or validate properties of geometric figures using appropriate tools and technology.

2.2.3 The student will use inductive or deductive reasoning.

Materials Needed

Worksheets and the tools of HSA geometric constructions (compass, straightedge, patty paper, Mira™, or mirrors).

Pre-requisite Concepts Needed

Students should have an understanding of the rules of geometric constructions.

Approximate Time

One 45-minute lesson and a follow-up for 20 minutes in a subsequent lesson.

Constructions, Sketches, and Drawings

Lesson Plan

Essential Questions

What is the difference between a construction, a sketch, and a drawing?

When is it appropriate to use a sketch instead of a drawing?

When is it appropriate to use a drawing instead of a construction?

When is it appropriate to use a construction instead of a drawing?

Warm-Up/Opening Activity

Define the terms *construction*, *sketch*, and *drawing*. When have you seen these words used? What has been the application of each of these words?

Start a K-W-L (What do you already **know**?, What do you **want to learn**?, What did you **learn**?) chart with the students to find out what they already know (correctly or incorrectly) about the similarities and differences between constructions, sketches, and drawings. Collect the information on the **KWL Chart**, completing the first two columns.

Development of Ideas

Use concept attainment with examples to build the definitions for each term using the warm-up as a starting point

Construction – Constructions of figures and lengths with only a compass and a straightedge as tools. Sometimes, the term “ruler” may be used instead of straightedge, but the accepted classical definition is for the use of a straightedge without any markings that could be used to measure.

Classical geometric constructions use only a compass and a straightedge as tools. For the High School Assessment, students may also use patty paper, Mira™, or mirrors.

Drawings – Drawings can use all of the tools listed for constructions and measurement tools can also be used to reflect relationships.

For items that ask the students to draw a geometric figure, students may use a compass, ruler, patty paper, Mira™, mirrors, and/or protractor. Measurement can be part of their strategy.

Sketch – A drawing that can be completed without the use of tools.

Constructions, Sketches, and Drawings

Development of Ideas (Continued)

Using the definitions, discuss each of the following questions with the class.

- Why are there differences between constructions, sketches, and drawings?
- What is the purpose of each?
- When is it better to use one instead of the other?
- What is the role of each?
- What real world applications does each have?

A key difference between a construction and a drawing is that in constructions all relationships are determined by the use of formal proof, not measured accuracy.

> To the Greeks, where the current formalized study of Geometry started, all knowledge was broken down into its most simple parts, which for constructions was the use of basic tools, the compass and the straightedge.
>
> To a mathematician, there is no "exact" measurement or "perfectly reliable" tool, only measurements and tools with less and less error. The key to constructions is that they can be proved to exhibit the relationships shown, not just drawn to show the relationships. That is why constructions and proof are so important in Geometry. Constructions are a way to visualize the formal structure of proof in classical Geometry.

Use the problems on **Drawing and Construction Examples** as examples of how HSA problems use the terms construction and draw. In the first problem, focus the student's attention on how students need to measure in order to solve the problem, indicating that it truly is a drawing. In the second problem, students will again note that measurements are important in determining the relationship, so that again it is a drawing activity, not a construction. In the third problem, students must justify their answer using properties of mathematics, not measurements, making it a true classical construction problem.

Answers:
1. 12 feet
2. No, there is no AAA Congruency.
3. Where the perpendicular bisector of $\overline{AB}$ meets line l. The reasoning can include SAS Congruency or if a point is on the perpendicular bisector it is equidistant from the endpoints of the segment.

Work with students to understand "Note: Figure not drawn to scale"

> Be sure that students do not measure when this phrase is in the problem. Use **"Figure Not Drawn to Scale" Examples** for examples of problems in which the information in the problem should be used, not any measured relationships from the diagram.

Answers: 1. G   2. H   3. 140 inches

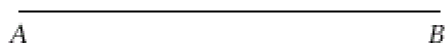
Constructions, Sketches, and Drawings

Development of Ideas (Continued)

Examine the difference between explanation and justification with constructions. Using the sample problems below, ask some of the students in the class to explain their answer, another group to justify their answer. **Explanation/Justification Examples** has the following examples for students to work on. Assign half of the class to work on the first bullet (explanation) and the other half to work on the second bullet (justification).

Sample 1 - HSA 2000 Public Release Question [excerpt]

Use the segment below to complete the following **construction**.



Using segment $\overline{AB}$, construct equilateral triangle *ABC*.

- Use mathematics to explain the process you used to construct the triangle. Use words, symbols, or both in your explanation.
- Use mathematics to justify that the triangle you constructed is an equilateral triangle.

An explanation could be:

With my compass I drew an arc equal to AB from points A and B. The 2 arcs intersected at point C. ΔABC *is equilateral.*

A justification of what they have done could be:

Since I used the same arc measure to construct $\overline{AC}$ *and* $\overline{BC}$, *AB = AC = BC. Therefore,* ΔABC *is equilateral* **or** *giving the measurement of each side and stating that all sides have the same measurement, therefore,* ΔABC *is equilateral.*

Sample 2

Draw an equilateral triangle with sides equal to 2.5 inches.

- Use mathematics to explain the process you used to draw the triangle. Use words, symbols, or both in your explanation.
- Use mathematics to justify your triangle is equilateral.

An explanation of what they have done could be:

I used my ruler to draw AB = 2.5 inches. Then I used my protractor to draw angles A and B to be 60 degrees. Point C is located where the sides of angles A and B intersect.

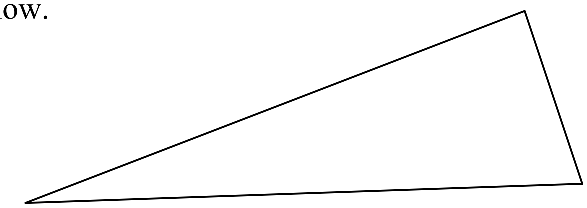
A justification could be:

I measured each angle. They each measured 60 degrees. I measured each side. They each measured 2.5 inches. Therefore, ΔABC *is equilateral.*

Development of Ideas (Continued)

Sample 3

Helen is designing a quilt pattern. Part of the design is a triangle of one fabric with a circle of a contrasting fabric inscribed in the triangle. The triangle is shown below.



Construct the inscribed circle.

- Use mathematics to explain the process you used to draw the circle. Use words, symbols, or both in your explanation.
- Use mathematics to justify that your circle is inscribed in the triangle.

An explanation of what they have done could be:

I constructed the angle bisectors of each angle. Their point of intersection is the center of the circle. To find the radius, I constructed the perpendicular from that center to one side. Using the center and the radius, I constructed the circle that is inscribed in the triangle.

A justification could be:

Since the point of concurrency of the angle bisectors of a triangle is the center of the inscribed circle, I constructed the angle bisectors. Since the inscribed circle is tangent to each side of the triangle, I found the perpendicular distance from the center to one side of the triangle. This gave me the radius of the inscribed circle. **Or** *The circle I constructed is tangent to all of the sides of the triangle, therefore, it is an inscribed circle.*

Have the students construct a square, draw a square, and sketch a square. Have the students explain the difference in each.

Follow-Up in the Next Lesson

Discuss with students the answers to the construct, draw, and sketch a square. Emphasize the differences between the three squares, referring to the definitions used from the previous lesson. Encourage the students to edit their work to include this type of information.

Follow-Up in the Next Lesson (Continued)

Complete the last column of the KWL Chart from the opening activity as a summary of what was learned in the lesson. Be sure that all items that the students desired to know have been instructed. This will connect the opening and closing of the activity.

Supplemental Activities/Resources

High School Assessment Public Release questions
  2000 Version – problems 8, 9, 31, and 38
  2001 Version – problem 30

"Construction vs Drawing on the High School Assessment" discussion paper from MSDE

Extension Activities

Have students explore the three classic impossible geometric constructions: trisecting an angle, squaring the circle, and doubling the cube.
  Discussion of these problems can be found at:
  http://mathforum.org/dr.math/faq/faq.impossible.construct.html

Use of computer technology to draw and construct geometric figures

Constructions, Sketches, and Drawings

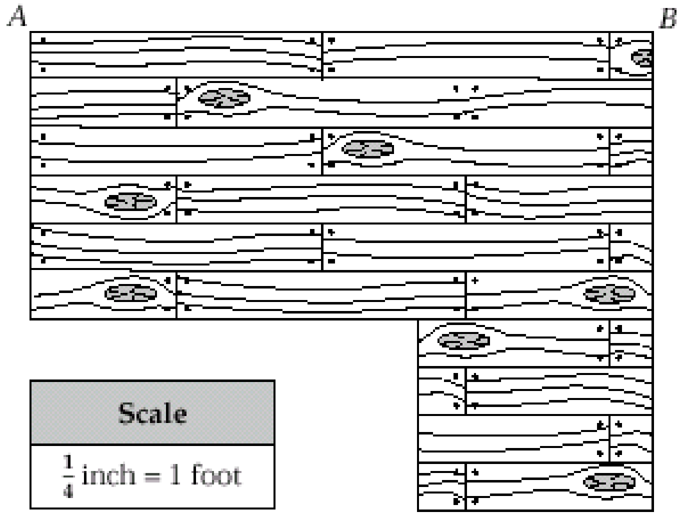
KWL Chart

	What Do You Already **Know**?	What Do You **Want to Learn**?	What Did You **Learn**?
Construction			
Sketch			
Drawing			

Constructions, Sketches, and Drawings

Drawing and Construction Examples

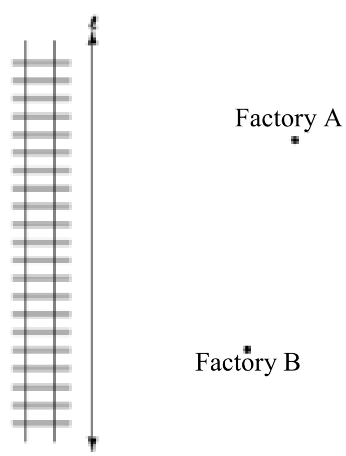
1. A scale drawing of a deck is shown below. (*HSA 2000 Public Release Question*)



On the actual deck, what is the length, in feet, of $\overline{AB}$?

2. Elizabeth draws a right triangle with angles of 52° and 38°. Complete the following: (*HSA 2000 Public Release Question*)
   - Draw a right triangle using these angle measurements. Label the measure of each angle.
   - Will any right triangle with angles of 52° and 38° be congruent to Elizabeth's triangle? Use mathematics to justify your answer.

3. Two factories are located near a railroad. There is a loading platform on line *l* equidistant from the two factories. (*HSA 2001 Public Release Question*)

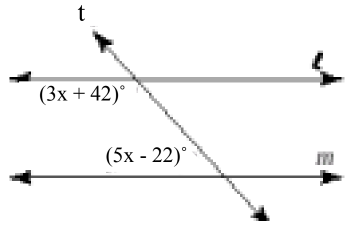


   - Use geometric constructions to determine the location of the loading platform. Label the loading platform with an *X*.
   - Use mathematics to justify your answer.

Constructions, Sketches, and Drawings

"Figure Not Drawn to Scale" Examples

1. Line *l* is parallel to line *m*. Line *t* is a transversal with angle measures as indicated below. (*HSA 2000 Public Release Question*)



Note: The figure is not drawn to scale

What is the value of *x*?

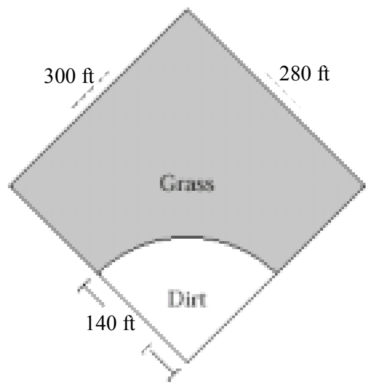
F 16

G 20

H 25

J 32

2. A worker at a new baseball park is mowing the shaded region of the rectangular field shown below. The dirt region is a quarter circle with a radius of 140 feet. (*HSA 2000 Public Release Question*)



Note: The figure is not drawn to scale

How many square feet of grass will the worker mow? Round the answer to the nearest square foot.

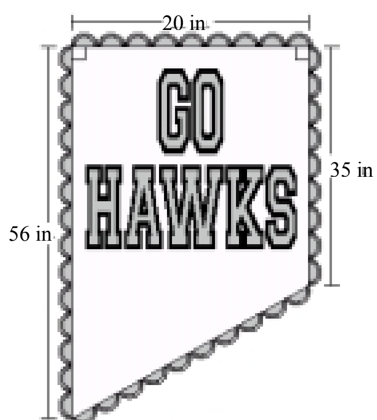
F 15,394 square feet

G 22,425 square feet

H 68,606 square feet

J 84,000 square feet

3. A school is purchasing trim to go around all four edges of the banner shown below. (*HSA 2000 Public Release Question*)



Note: The figure is not drawn to scale.

How many inches of trim should be purchased?

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Explanation/Justification Examples

For each example, answer the bullet that the teacher asks you to complete.

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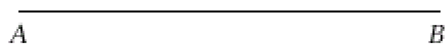
Use the segment below to complete the following **construction**.



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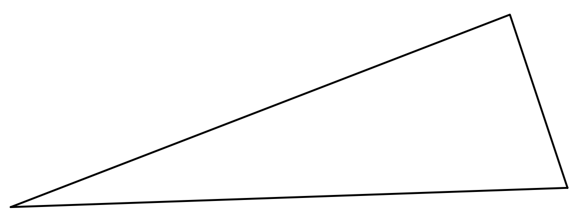

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