The Pythagorean Theorem, the Distance Formula, and Slope

Objective

For the students to understand the connection between the Pythagorean Theorem, the distance formula, and slope.

Core Learning Goals

2.1.2 The student will identify and/or verify properties of geometric figures using the coordinate plane and concepts from algebra.

2.3.1 The student will use algebraic and/or geometric properties to measure indirectly.

Materials Needed

Worksheets and overheads

Pre-requisite Concepts Needed

The Pythagorean Theorem can be instructed prior to the lesson or as part of the warm-up activity.

Approximate Time

One 45-minute lesson plus 10 minutes in a subsequent lesson.

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Lesson Plan

Essential Questions

What is the connection between the Pythagorean Theorem and the distance formula?

How are the distance formula, the Pythagorean Theorem, and slope interrelated?

Warm-up/Opening Activity

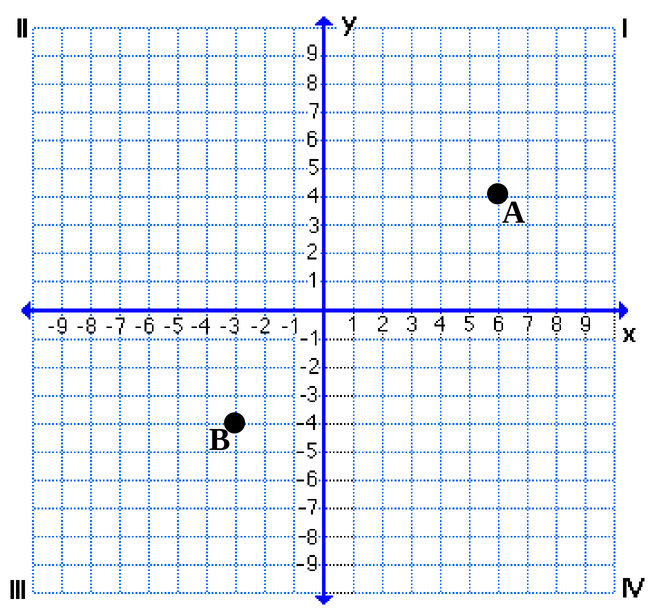
In their own words, have students write the Pythagorean Theorem. Discuss with the students what needs to be stated in the Pythagorean Theorem (it must be a right triangle and the longest side is always the hypotenuse). This pre-assessment can be used to determine the background knowledge of the students and what instruction may be needed to begin the lesson.

Answer: The Pythagorean Theorem occurs only in a right triangle, the square of the length of the hypotenuse (the longest side) is equal to the sum of the squares of the lengths of the legs of the right angle.

Development of Ideas

The distance formula is derived from the Pythagorean Theorem. What is the distance between points A and B? We can use the Pythagorean Theorem to find the distance. Using an example of two points and then expanding to a more generalized situation, students can see how the distance formula can be determined. What is the distance between points A and B?

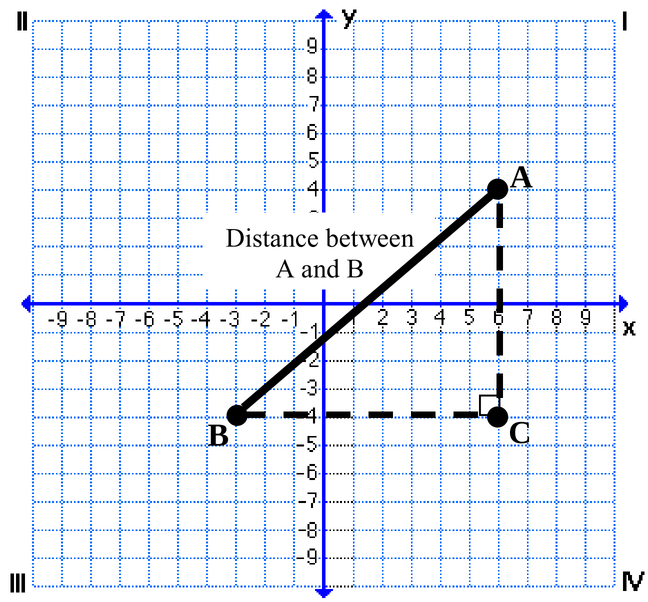
Overhead 1



Development of Ideas (Continued)

One way to determine the distance from point A to point B is to use the Pythagorean Theorem. The point A is at (6, 4) and the point B is at (-3, -4). Drawing lines to create a right triangle gives us:

Overhead 2



With the creation of the right triangle, the length of the hypotenuse ($\overline{AB}$) is the distance from point A to point B. The length of $\overline{AC}$ is the difference in the y-coordinates of the points, or 4 – (-4), which is 8. The length of $\overline{BC}$ is the difference in the x-coordinates of the points, or 6 – (-3), or 9. Using the Pythagorean Theorem, we get:

$$a^2 + b^2 = c^2$$

$$9^2 + 8^2 = c^2$$

$$81 + 64 = c^2$$

$$145 = c^2$$

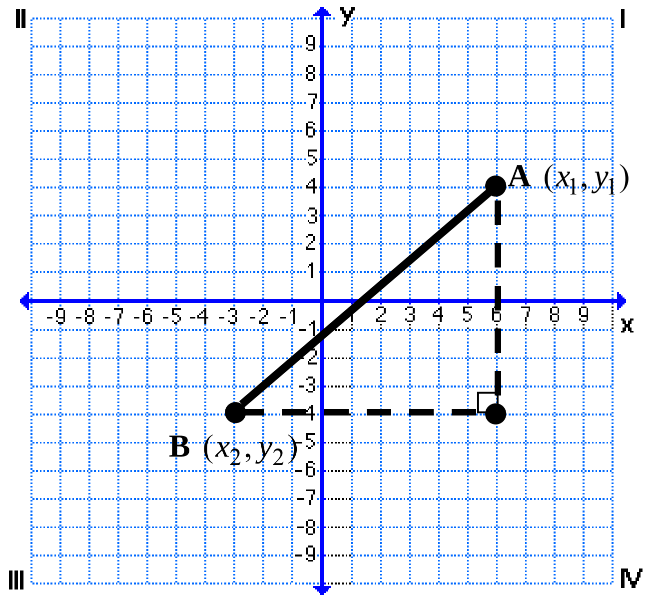
$$\text{so that } c \approx 12.04$$

So that the length of $\overline{AB}$ is 12.04, meaning that the distance from A to B is about 12.04 units.

We can now generalize this process for any two points. We can generalize the points A and B to be at any coordinate, for example A can be at (x_1, y_1) and B can be (x_2, y_2).

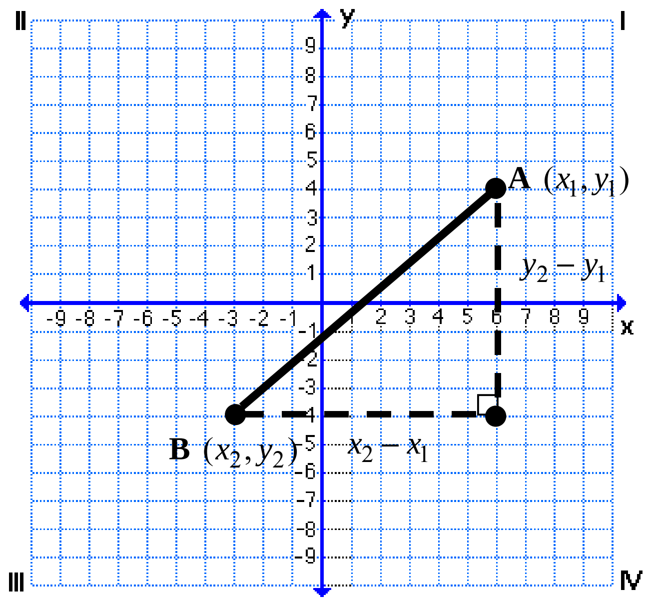
Development of Ideas (Continued)

Overhead 3



Since the points are identified in generic terms, we can see the legs of the right angle as changes in the arbitrary coordinates, or $y_2 - y_1$ for the vertical length and $x_2 - x_1$.

Overhead 4



Now, using the Pythagorean Theorem,

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = (\text{distance})^2$$

Development of Ideas (Continued)

Now, solving for distance by taking the square root of both sides,

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\ ,$$

which is the distance formula for any two points in the coordinate plane.

Looking back at the earlier example, with point A at (6, 4) and point B at (-3, -4), then,

$$\text{distance} = \sqrt{(-3-6)^2 + (-4-4)^2} = \sqrt{(-9)^2 + (-8)^2} = \sqrt{81+64} = \sqrt{145} \approx 12.04$$

Note that the distance is the same no matter the derivation.

In this first example, it is clear that the Pythagorean Theorem and the distance formula tell us only limited information, the distance between the two points. We can gain additional information from these points, including the slope between the points. Using the same points, the slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-4}{-3-6} = \frac{-8}{-9} = \frac{8}{9}$$

This indicates that the direction between these two points follows a line with a slope of $\frac{8}{9}$ and is a way that students can see how the same idea, the difference of *x* and *y* coordinates can be used in different ways to find different characteristics of two points. This connection between distance and direction can preview the study of vectors, which give both direction and distance.

Worksheet: **HSA Practice 1**

Answers: 1. C 2. H 3. C 4. H

Worksheet: **HSA Practice 2**

Answers: The distance is $\sqrt{125} \approx 11.18$, student reasoning will vary, but the justification matching the reasoning is the key.

The slope is $-\frac{1}{2}$

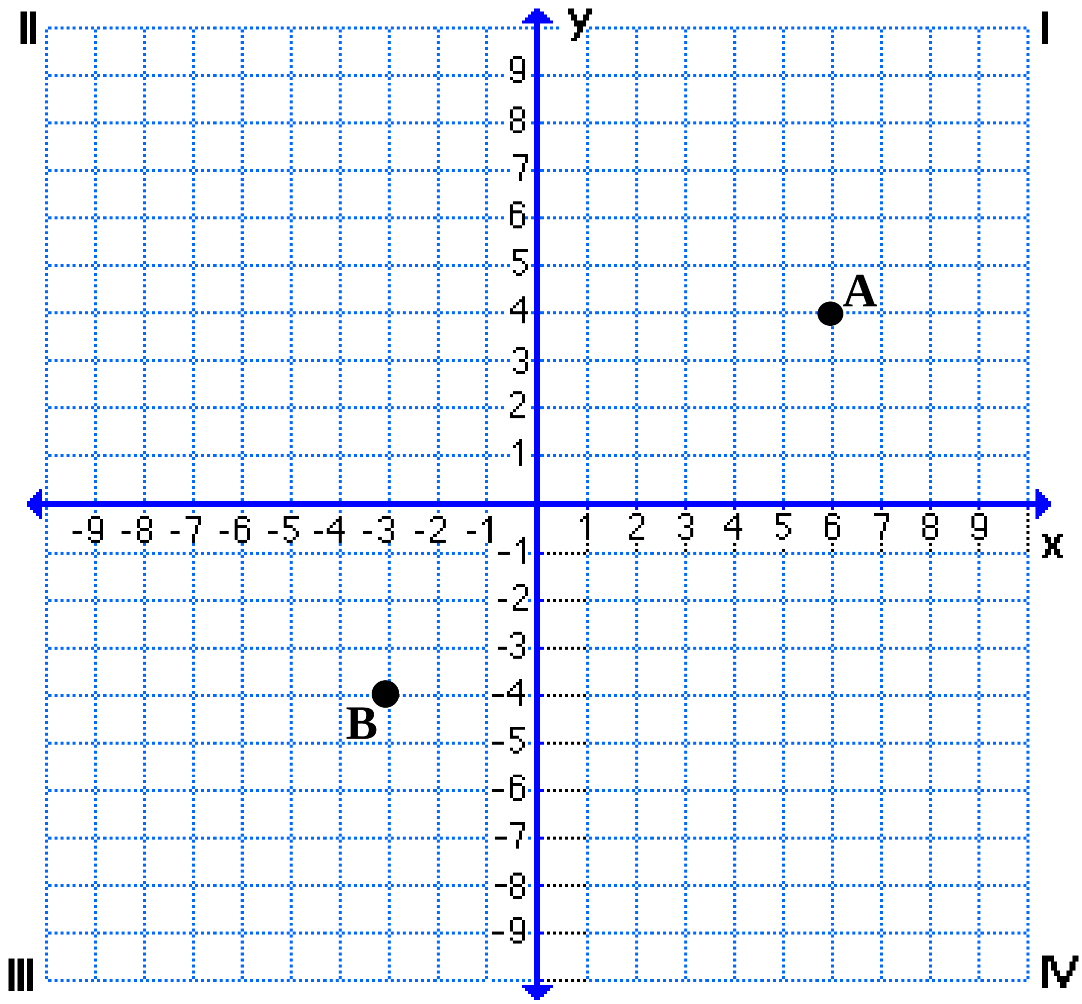
Follow-Up Activity

Discuss the worksheet with the students in the next lesson. Focus on student explanations of which process they feel is best, not necessarily the selection.

Supplemental Activities

High School Assessment Public Release questions
2000 Version – problems 11 and 14 2001 Version – problems 15, 20, 21, 37, 41

Proving the Distance Formula 1



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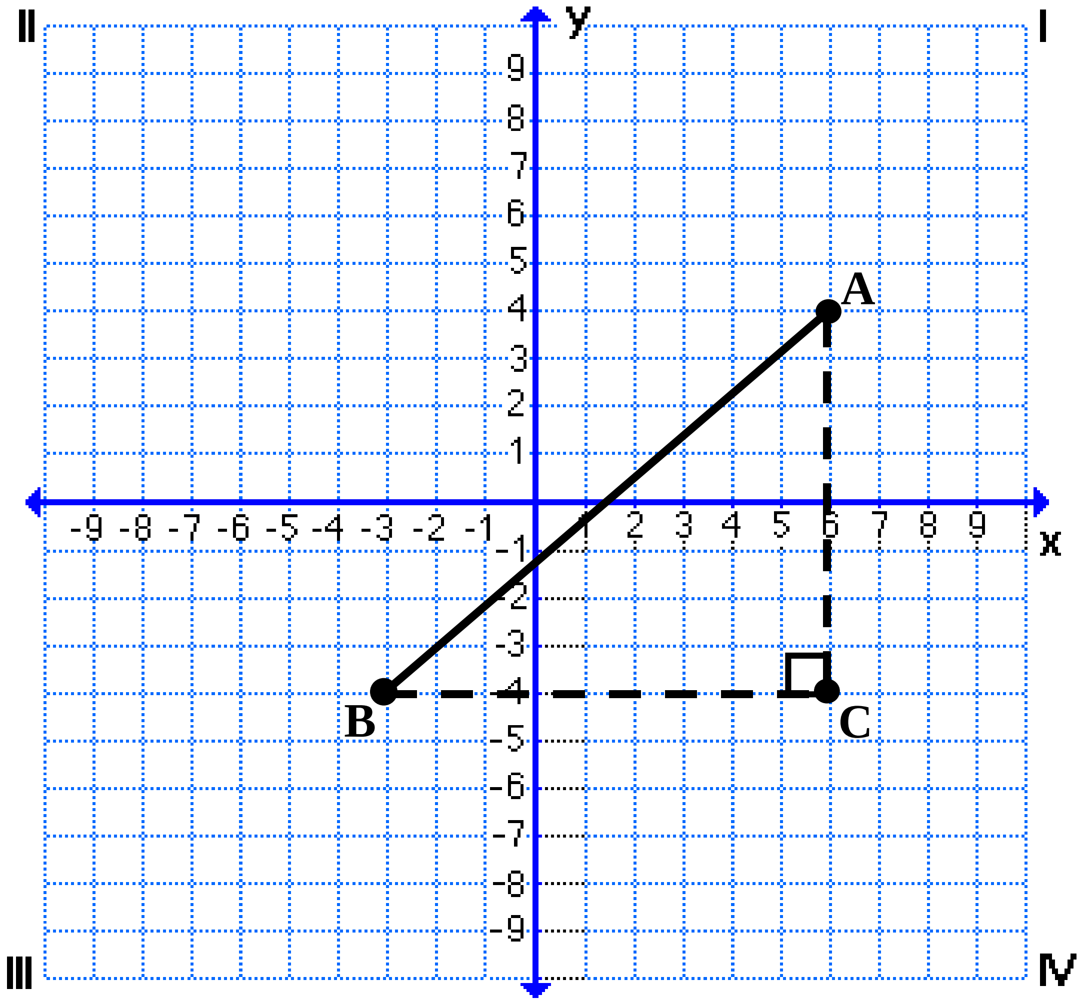
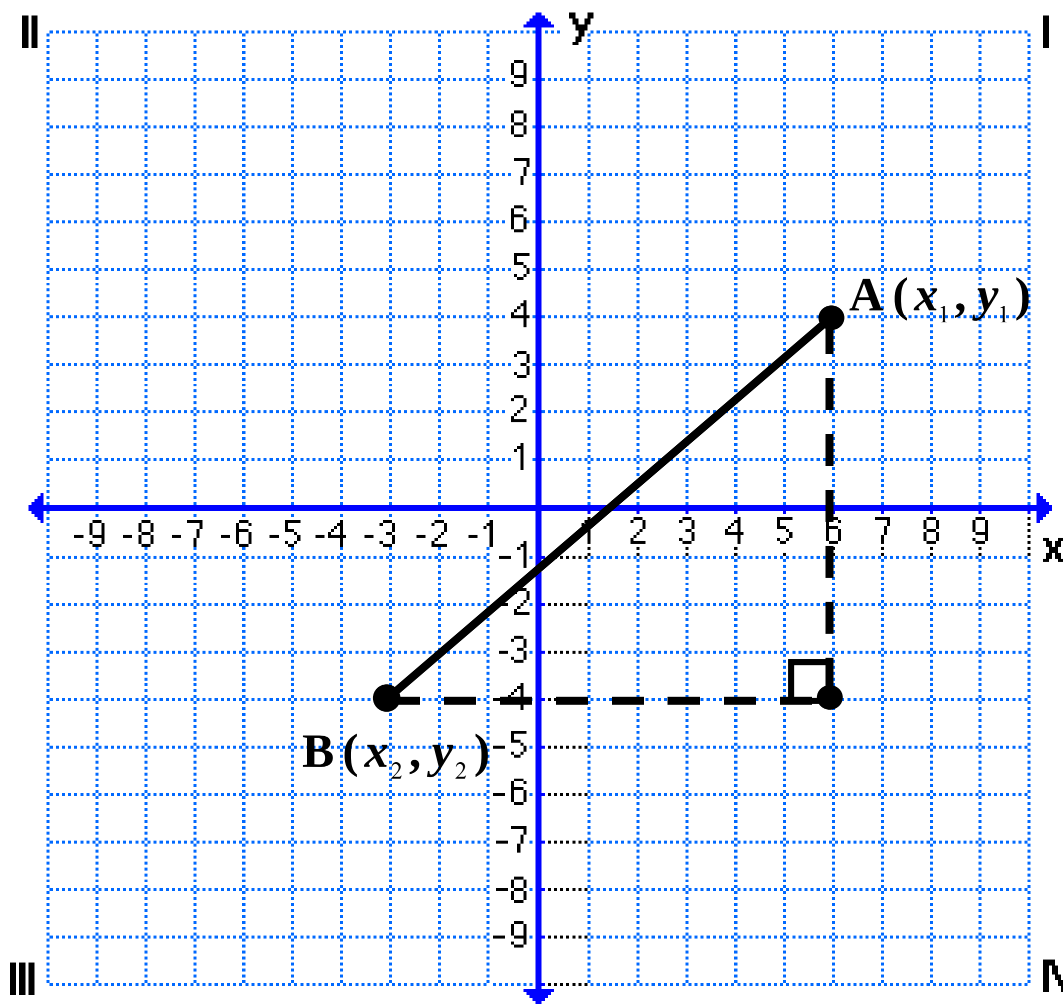
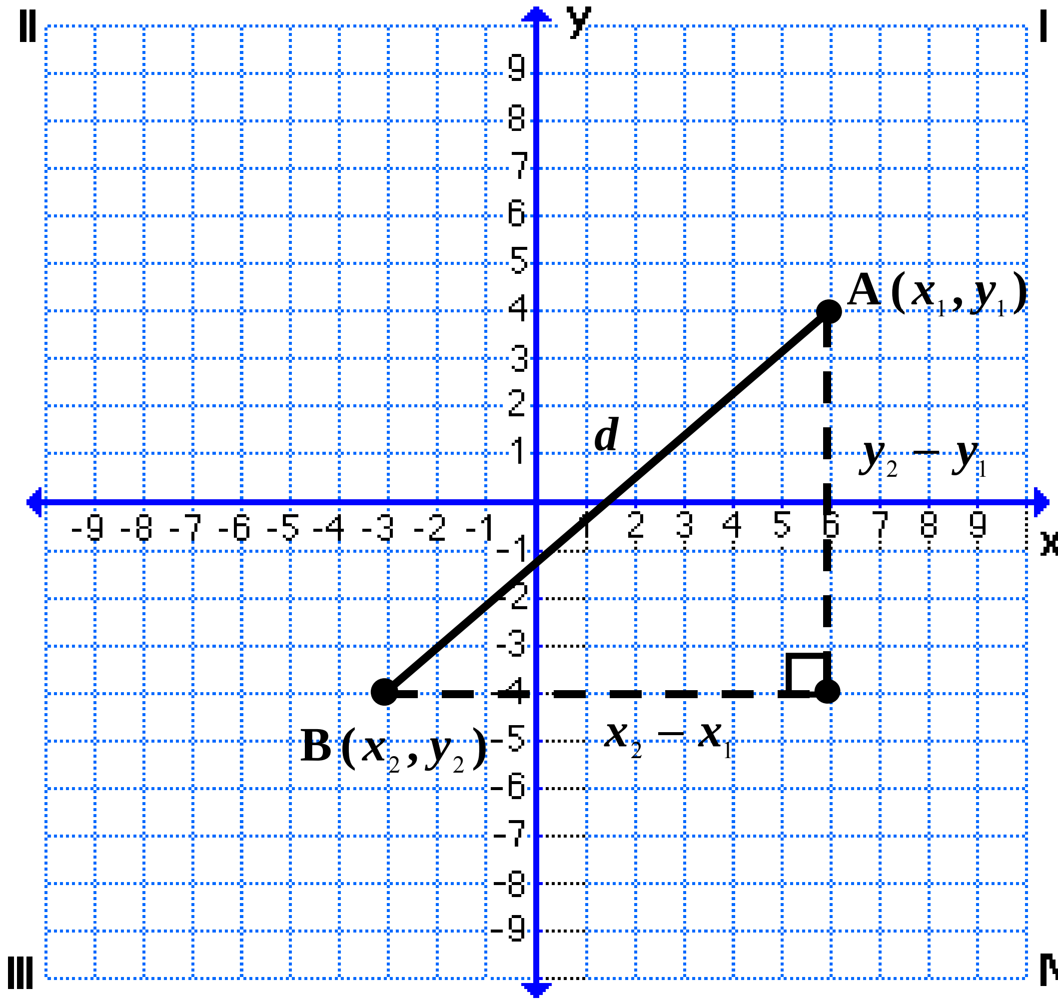
Proving the Distance Formula 2



Proving the Distance Formula 3



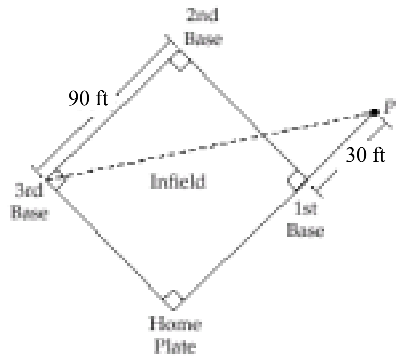
Proving the Distance Formula 4



The Pythagorean Theorem, the Distance Formula, and Slope

HSA Practice 1

1. A diagram of a baseball field is shown below. The infield is a square that measures 90 feet on each side. (*HSA 2001 Public Release Question*)



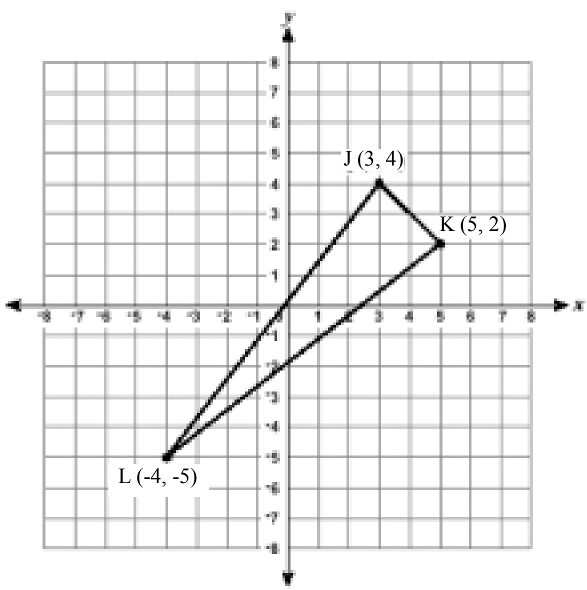
<u>Note:</u> The figure is not drawn to scale.

A player threw a ball from point *P* to third base. How far did the player throw the ball? Round the answer to the nearest foot.

A 79 feet | B 127 feet

C 150 feet | D 210 feet

2. Triangle *JKL* is shown on the grid below. (*HSA 2000 Public Release Question*)



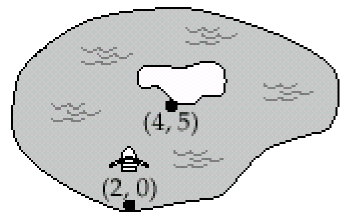
What is the length of $\overline{KL}$? Round the answer to the nearest tenth of a unit.

F 3.2 units | G 4.0 units

H 11.4 units | J 11.7 units

HSA Practice 1 (Continued)

3. A lake is shown below. An island is located at (4, 5). A boat travels in a straight line from (2, 0) to the island. (*HSA 2001 Public Release Question*)



How far does the boat travel? Round the answer to the nearest tenth of a unit.

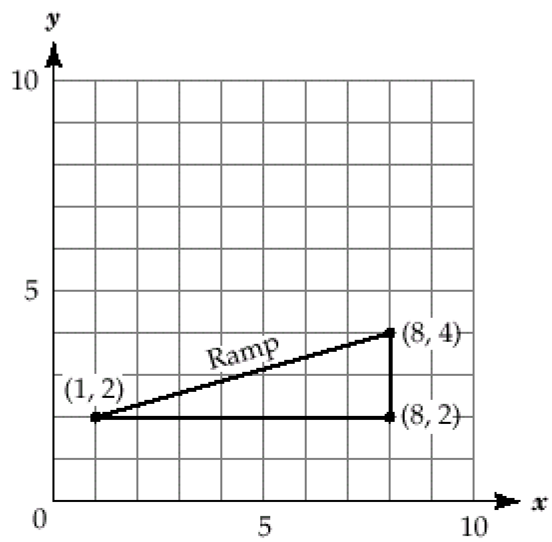
A 3.3 units

B 3.7 units

C 5.4 units

D 7.8 units

4. An architect is designing a ramp for delivery trucks. A drawing of the ramp is shown on the grid below. (*HSA 2001 Public Release Question*)



What is the slope of the ramp?

F $-\frac{7}{2}$ G $-\frac{2}{7}$

H $\frac{2}{7}$ J $\frac{7}{2}$

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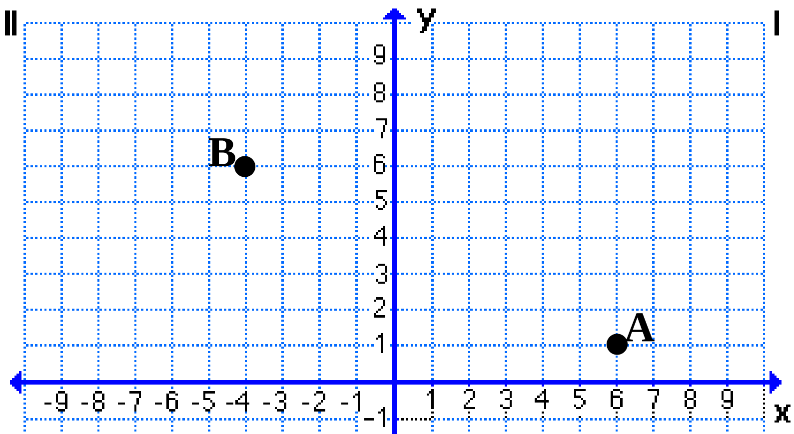
HSA Practice 2

Determine the distance between the two points, using both the Pythagorean Theorem and the distance formula.



Pythagorean Theorem	Distance Formula

Which method do you think is better to determine the distance between two points? Use mathematics to justify your answer.

Determine the slope between the two points.