Goal 3: Data Analysis & Probability

Calculating Theoretical & Experimental Probability

OBJECTIVE	CORE LEARNING GOALS
• Students will collect data by conducting a survey of fellow classmates. • Students will investigate the relationship between estimates obtained from samples and theoretical probabilities.	3.1.3 The student will calculate theoretical probability or use simulations or statistical inferences from data to estimate the probability of an event.
DRILL A random sample of 150 people are asked if they own dogs, and 57 of them say yes. What would you estimate the percentage of dog owners to be in the general population? *(The sample proportion is* $57/150 = 0.38$*. Since the sample is random, the percentage of the population owning dogs should be close to 38%.)*	MATERIALS Activities: "Collecting Your Own Data," "From the Experimental to the Theoretical," and "Calculating Theoretical Probabilities" Answer Keys Coins, dice, spinners, or other similar devices

CALCULATOR SKILLS

None

ACTIVITIES

1. **Drill.** Students will estimate the population percentage from the results of a sample survey.
2. **Exploration.** The first activity, "Collecting Your Own Data," helps students grasp how survey results can be used to make predictions regarding the population. The activity also gives students experience with experimental probability. The next activity relates probabilities determined from sample results to those that we would expect in the long run by combining all possible samples. In "From the Experimental to the Theoretical" it may be necessary to combine students' results to show that experimental probabilities will approach the theoretical values in the long run or after many tosses.
3. **Class Discussion.** Students need to understand that experimental probabilities vary because they depend on the results of samples. However, theoretical probabilities are determined by considering all possible outcomes and how often they occur in the long run. For example, the theoretical probability of tossing a coin and landing on heads is 50%. This value means that it the long run, after many, many tosses, we can expect a coin to land on heads approximately half the time. It does not mean that the coin will land on heads once in two tosses.
4. **Additional problems.** The last activity gives students practice calculating theoretical probabilities.

ASSESSMENT	HOMEWORK
Summary Questions: Randy wanted to determine his chances of making a basket in a basketball game. In 15 free throw attempts he made 9 of the shots. 1. What proportion of shots did Randy make? *(sample proportion:* $9/15 = 0.60$*)* 2. Do you think that this value is a good estimate for Randy's probability of making a basket? Use mathematics to justify your answer. *(It is difficult to tell whether or not 0.60 is a good estimate because Randy attempted only 15 free throws. If he had attempted 1500 free throws and made 900 of them, we could be more confident in the estimate.)*	"Calculating Theoretical Probabilities"

Collecting Your Own Data

Survey the students in your class to see how many have ever been to an Orioles game and how many have been to a Ravens game. Record the numbers in the following table. For example, if a student has never been to a Ravens game but has been to an Orioles game, make a tally mark in the top right box of the table.

		Have you ever been to a Ravens game?	
		Yes	No
Have you ever been to an Orioles game?	Yes		
	No		

Because the information above was not gathered using simple random sampling techniques, the results of your data collection will *not* be representative of the entire student body. However, for the sake of this exercise, assume that the data was collected from a simple random sample of students in your school.

Suppose another student is randomly selected from the student body. Use the results of the survey to find the following probabilities.

1. Based on your survey, what is the probability that the student has been to an Orioles game?

2. Based on your survey, what is the probability that the student has been to a Ravens game?

3. Based on your survey, what is the probability that the student has been to both an Oriole and a Ravens game?

4. Consider a group of 100 students (selected randomly) from your school whom you did not interview.

    a. How many students would you predict have *not* been to an Orioles game? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

    b. How many students would you predict have *not* been to a Ravens game? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

From the Experimental to the Theoretical

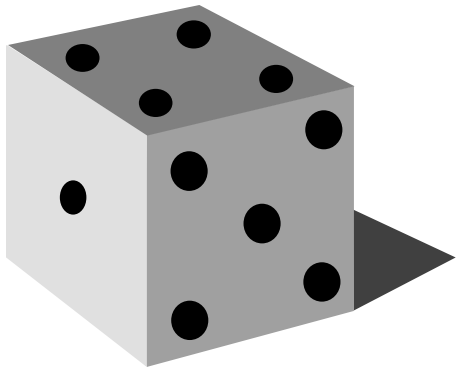
In this activity you will explore the relationship between theoretical and experimental probability. In the previous activities we explored **experimental probability**. We estimated probabilities using the results of data collected from samples of a larger population. **Theoretical probability** requires that we consider all possible outcomes. We determine the likelihood of an event by relating the number of favorable outcomes to the total number of outcomes.

Probability of event *E*,

$$P(E) = \frac{\text{number of outcomes making up event } E}{\text{total number of outcomes}}.$$

Suppose we collected many samples from the same population. The experimental probabilities determined from these samples should be close to the theoretical probabilities in the long run.

Consider the 6-sided number cube below. The six faces will appear at about the same rate when the cube is tossed, so the six possible outcomes should be equally likely.



1. Calculate the following theoretical probabilities.
   a. What is the theoretical probability that the cube will land on 6 when tossed?

   b. How often will the cube land on an even number?

   c. If you toss the cube one time, what is the likelihood that it will land on either 1 or 2?

   d. Suppose you toss the cube eight times and each time it lands on 3. What is the likelihood of getting a 3 on the next toss?

2. Calculate the following theoretical probabilities.

   a. P(landing on an odd number)

   b. $P(4)$

   c. $P(3,\ 4,\ 5,\ \text{or } 6)$

   d. $P(8)$

3. Obtain a number cube from your teacher and toss it 30 times. Record your results in the frequency table below.

Outcome	1	2	3	4	5	6	Total
Frequency							30

4. Your thirty tosses represent a sample from the population of all possible tosses. Estimate the following experimental probabilities using your sample results.

   a. likelihood of landing on an odd number

   b. likelihood of landing on 4

   c. likelihood of landing on 3, 4, 5, or 6

   d. likelihood of landing on 8

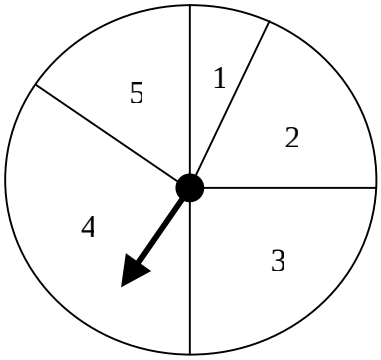
5. Compare the experimental probabilities in exercise 4 to the theoretical probabilities you determined in exercise 2.

6. Suppose you tossed the cube 10,000 times. How would your experimental estimates compare to the theoretical probabilities?

Calculating Theoretical Probabilities

In the previous activity, you determined the likelihood of events associated with tossing a 6-sided number cube, where all faces on the cube were equally likely. In this activity, you will calculate the probability of events that are not equally likely.

Consider the spinner below. Notice that the spinner is divided into five parts and that the parts have different areas.



The following table gives the probability distribution for the spinner.

Outcome	1	2	3	4	5
Probability	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{6}$

1. The spinner must land on one of the 5 outcomes, so the probabilities in the table should add
up to 1 or 100%. Use mathematics to justify that the sum of the probabilities is 1.

2. Determine the following probabilities using the values in the table. Use mathematics to explain how you determined your answers. Use words, symbols, or both in your explanations.

   a. What is the probability of landing on an even number?

   b. What is the probability of landing on an odd number?

   c. What is the probability of landing on an even or an odd number?

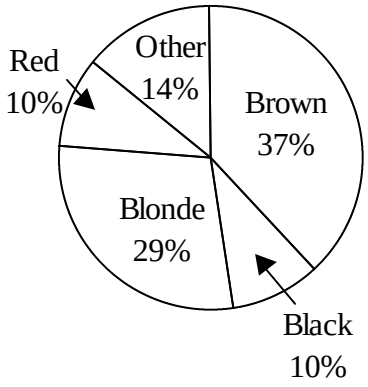
3. Determine the following probabilities using the values in the table. Use mathematics to explain how you determined your answers. Use words, symbols, or both in your explanations.

   a. What is the probability of landing on 2 or 3?

   b. What is the probability of landing on 6?

The pie chart (or circle graph) below gives the distribution of hair colors for twenty-one students in a particular math class.

Hair Color



4. Suppose a student in this math class is chosen at random. Use the information in the pie chart to determine the following probabilities.

   a. What is the probability that the student has blonde hair?

   b. What is the probability that the student chosen has red or black hair?

   c. What is the probability that the student has curly hair?

   d. What is the probability that the student's hair color is *not* brown?

5. Suppose that the students in the math class represent a simple random sample of all students in the school.

   a. What is the probability that a randomly selected student in the school has blonde hair?

   b. What is the probability that a randomly selected student in the school has hair color other than blonde, brown, black, or red?

Many board games involve tossing 6-sided number cubes. Suppose you are playing a game where two number cubes are tossed and the values on the faces of the cubes are added together. The possible outcomes in this game would consist of the possible sums of the two faces of the cubes.

It will be helpful in this activity to list all possible combinations of the two cubes, so that the outcomes are equally likely. Look at the table below.

Faces	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

There are thirty-six possible combinations of the two cubes. Suppose we wanted to calculate the probability of getting a sum of 4. Three of the thirty-six combinations yield a sum of 4, so $P(4) = 3/36$.

6. Complete the following table by calculating the probability of each possible sum.

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability			$\frac{3}{36}$								

7. Determine the probability of each event using the above table. Use mathematics to explain how you determined your answers. Use words, symbols, or both in your explanations.

   a. What is the probability of getting a sum greater than 9?

   b. What is the probability of getting a sum less than or equal to 5?

   c. What is the probability of getting a sum that is an even number?

   d. What is the probability of getting a sum equal to 5, 6, or 7?

   e. What is the probability of getting a sum of 1?

8. Suppose that you observed a sum of 8 on the last three tosses. What is the probability that your next toss will yield a sum of 8? Use mathematics to justify your answer.

Collecting Your Own Data
Answer Key

Answers for this activity will vary. Consider the sample results below.

		Have you ever been to a Ravens' game? Yes	Have you ever been to a Ravens' game? No
Have you ever been to an Orioles' game?	Yes	5	13
	No	5	7

1. Answers will vary. For the sample results above: $\frac{18}{30} = 0.60$.

2. Answers will vary. For the sample results above: $\frac{10}{30} = 0.333$.

3. Answers will vary. For the sample results above: $\frac{5}{30} = 0.167$.

4. a. Answers will vary. For the sample results above: $\frac{12}{30}(100) = 40$.

   b. Answers will vary. For the sample results above: $\frac{20}{30}(100) = 66.6$ or approximately 67.

From the Experimental to the Theoretical
Answer Key

1. a. $\frac{1}{6} = 0.167$

   b. The cube should land on an even number in about half of the tosses.

   c. $\frac{2}{6} = \frac{1}{3} = 0.333$

   d. $\frac{1}{6}$, the outcomes of the prior tosses do not affect the outcome of the ninth toss. We call this "independence."

2. a. $\frac{1}{2} = 0.50$

   b. $\frac{1}{6} = 0.167$

   c. $\frac{4}{6} = \frac{2}{3} = 0.667$

   d. 0

3. Answers will vary. The long-run frequencies (theoretical) are shown below.

Outcome	1	2	3	4	5	6	Total
Frequency	5	5	5	5	5	5	30

4. a. $\frac{15}{30} = \frac{1}{2} = 0.50$

   b. $\frac{5}{30} = \frac{1}{6} = 0.167$

   c. $\frac{20}{30} = \frac{2}{3} 0.667$

   d. 0

5. The experimental probabilities should be somewhat close to the theoretical probabilities, although some may be further off than others. Combining student/group results should yield experimental probabilities even closer to what we'd expect in the long run.

6. After 10,000 tosses the experimental probabilities should be extremely close to the theoretical probabilities.

Calculating Theoretical Probabilities
Answer Key

1. $\frac{1}{12}+\frac{1}{6}+\frac{1}{4}+\frac{1}{3}+\frac{1}{6} = \frac{1}{12}+\frac{2}{12}+\frac{3}{12}+\frac{4}{12}+\frac{2}{12} = \frac{12}{12} = 1$

2. a. $\frac{1}{6}+\frac{1}{3} = \frac{1}{6}+\frac{2}{6} = \frac{1}{2} = 0.50$

   b. $\frac{1}{12}+\frac{1}{4}+\frac{1}{6} = \frac{1}{12}+\frac{3}{12}+\frac{2}{12} = \frac{6}{12} = \frac{1}{2} = 0.50$

   c. $0.50 + 0.50 = 1$

3. a. – – — — —

   b. The spinner will never land on 6 because 6 is not a value for one of the parts of the spinner.

4. a.

   b.

   c. There is not enough information to determine this probability.

   d.

5. a.

   b.

6.

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	—	—	—	—	—	—	—	—	—	—	—

7. a. ——— — –

   b. ——— — —

   c. ——— — –

   d. ——— — —

   e.

8. — , the outcomes of the prior tosses do not affect the outcome of the ninth toss. We call this “independence.”