Goal 3: Data Analysis & Probability

Introduction to Measures of Central Tendency and Variability

OBJECTIVE	CORE LEARNING GOALS
The students will be able to calculate the mean, median, mode, range, and interquartile range for a set of data; create a box and whisker plot using paper and pencil and technology.	3.1.2 The student will use the measures of central tendency and/or variability (mean, median, mode, range, interquartile range, quartile) to make informed conclusions.
DRILL	MATERIALS " How Much is a Handful"; "Box and Whisker Plots and the Graphing Calculator"; "Box and Whisker Plots"; Graphing Calculator, manipulatives (see below)

CALCULATOR SKILLS

Enter data in a list; calculate the mean, median and range; and create a box and whisker plot

ACTIVITIES

1. **Drill**.
2. **Introduction -** How much is a handful? What if we wanted to add a new measure to the standard units of measure (like a foot, inch, meter, etc) called a handful? How would it be defined?

   Background: The International System of Units is a modernized version of the metric system, established by international agreement, that provides a logical and interconnected framework for all measurements in science, industry, and commerce. It has seven base units upon which all other units of measure are derived. They are Meter (length), Second (time), Kilogram (mass), Kelvin (temperature), Ampere (electric current), Candela (luminous intensity), and Mole (amount of substance). Each base unit is standardized. For example, a meter is defined to be the distance traveled by light in a vacuum in 1/29979245 of a second. The kilogram is the only base unit still defined by a physical object. The standard for a kilogram is a cylinder of platinum-iridium alloy kept by the International Bureau of Weights and Measures at Paris. (Source: Time Almanac 2000, 1999 Family Education Company, Inc.)

   In this lesson students will investigate the amount which constitutes a "handful". Using snap cubes, popcorn, beans, pennies, or another manipulative students will determine what makes a handful. Snap cubes (or "Unifix" cubes) work particularly well because they can be snapped together and students can compare stacks without actually having to count them.

3. **Class Exploration -** Show the manipulatives to the students and ask students to predict the number they think would be a handful and record their prediction on the student sheet. Students may ask whose hand?, which means they've already started to figure out the problem. However, ask them to make their prediction on their hand.

   ** Record all data on the board or an overhead projector**

After students record their prediction, ask students to each grab a handful of manipulatives, count them, and record the number on their sheet. Next, ask students to go to the front of the room and arrange themselves in order (least to greatest or greatest to least) standing shoulder to shoulder with students with the same number of manipulatives standing one in front of the other. (If you use snap cubes you may ask students not to talk while doing this portion because they can compare their stacks). Which student has the highest number? Which student has the lowest? Subtract these two numbers and ask the students if they know what this number is called. If not, introduce the term "range". We also call these numbers the extremes of the data.

Which group has the most people? (There may be the same number in two or more groups – this is okay. Ask students how they would decide which number has the most. Hopefully they will agree that it's a tie). Ask students if they know what this number is called, if not introduce the term "mode". Have students who have the same number of manipulatives stand side by side so that all students are standing shoulder to shoulder. Which student(s) is/are in the middle of the line? Ask students to step back two at a time, one from each end at your call. (Example: students are numbered 1, 2, 3, .. 30 consecutively. 1,30 step back then 2,29, then 3,28, etc.) The student (or students) who are remaining will represent the middle. If two students are in the middle, a discussion should take place to decide how the middle number will be decided upon. (the middle or average of the two numbers). Ask students if anyone knows what this number is called (the median) if not, introduce this term.

If only one student was in the middle, have this student remain standing forward. If two students were in the middle, then the median is between them and each of the two "middle" students step back. Since the median is the middle, then the students have been divided in half. Have the students in each half find the middle of themselves. (Following the same instructions as above – if two students are in the middle, then the students find the middle of those numbers). Half of a half would be one-quarter and these new "middle" numbers are called quartiles. Usually called an upper quartile and a lower quartile. Ask students which numbers were the extremes. These numbers (five-number summary of data) are the values that students will use to create a box and whisker plot.

Tell students to be fair, you really think each of them should have the same amount, therefore all students will put their handful in one place (like a bag or box) and then they will be redistributed so that each person has exactly the same amount. (Let's say the lowest handful was 5 and the highest was 11, where should we start? By giving everyone 1? Certainly not. Could we give everyone 3?4?5?7?11? Ask students these questions in reference to the specific data for the class.) Hopefully students will have some reasonable estimate for this number. Give out all of the manipulatives until each student has the same amount. If it does not work out evenly, then discuss how you would need to divide the remaining pieces so that everyone has the exactly the same amount. Ask students if they know what this number is called, if not introduce the term "mean" or as it is often called "average". For our purposes, the students should be using the term mean.

Have students return to their seats and record the class information for the data on their paper. Show students how to create a box and whisker plot for the data using the extremes, quartiles, and the median. **Show the students how a box and whisker plot can be drawn on a graphing calculator and how the five-number summary can be found on the calculator. The activity "Box and Whisker Plots and the Graphing Calculator" may be used as an additional class activity to practice using the graphing calculator to draw boxplots.**

.

4 **Questions for Discussion** - These questions may be discussed orally or in writing or as a whole class, in groups, or individually.

a. Which measure of central tendency (mean, median, or mode) best describes the amount in a "handful" for this class? Discuss the advantages and disadvantages of each.
Answers will depend on the data. Generally, if there is a really low or really high handful compared to the rest of the data, then the median would be a better measure because it is not affected by the "outliers". If many students had the same handful, then the mode might be the best measure. In the absence of outliers, mean may be the best measure.

b. What if I let a 3 year old come in and grab a handful and added his/her data to the class? What would this do to the mean? Median? Mode? Range? Would adding this data to the classes change your answer in question #1?
Letting a 3 year old grab a handful would probably create an outlier. The mean would decrease, the median would probably be unaffected, the mode would not be affected, and the range would increase. If you chose the mean as the best measure in question #1, then adding this data may change your mind to mode.

c. Justin came into the class and grabbed a handful. The mean for the class (including Justin's handful) increased, what must be true about the number in Justin's handful? *Justin's handful must be greater than the original mean number if the mean increased.*

d. Mary and Brenda joined the class and each grabbed a handful. The median for the class (including Mary and Brenda's data) did not change. What must be true about the number in Mary and Brenda's handful?
Either Mary and Brenda both grabbed a handful that was equal to the median for the class or one grabbed more than the median and one grabbed less, therefore not changing the median.

5. **Closure –**What are the measures of central tendency we discussed today?
Mean, median, and mode

How do you find the mean for a set of data? Median? Mode? (by hand and on a calculator)
Mean - add all of the numbers and divide by how many numbers you have.
Median - order the numbers from least to greatest then find the middle number.
Mode - determine the most frequent number.
Using the calculator - this depends on which calculator you are using.

What are the measures of variability that we discussed?
Range and interquartile range

How do you find the range? Quartiles? Interquartile range?
Range- subtract the lowest number (lower extreme) from the highest number (upper extreme).
Quartiles - after finding the median, find the median of each half of the data.
Interquartile range -find the difference between the two quartiles.

6. **Extension** - If possible, collect data from several classes and compare the classes the next school day. Have students draw conclusions about which class has the best or most typical handful. Comparing box and whisker plots among the classes would be a valuable exercise.

ASSESSMENT	HOMEWORK
Drill Class discussion	"Box and Whisker Plots"

Student Worksheet: How Much is a Handful?

1. Look at the manipulative your teacher has shown you, predict how many will be a handful?

2. My handful: Prediction _____ Actual _____

3. Record the number for each "handful" for your class in the table below.

Class Data:

4. Record the measures of central tendency and variance for your class data below.

Mean _____ Median _____

Mode _____

Lower Extreme _____ Upper Extreme _____

Range _____

Lower Quartile _____ Upper Quartile _____

Interquartile Range _____

5. Box and Whisker Plot:

Box and Whisker Plots and the Graphing Calculator

The table below shows the average monthly high temperatures in Columbus, Ohio and San Francisco, California.

Month	Columbus, OH	San Francisco, CA
Jan	35	56
Feb	38	59
Mar	49	60
Apr	62	61
May	73	63
Jun	81	64
Jul	84	64
Aug	83	65
Sep	77	69
Oct	65	68
Nov	51	63
Dec	39	57

Construct a box and whisker plot for the average monthly high temperatures in Columbus, Ohio and San Francisco, California on a graphing calculator using the following procedure.

1. Enter the Columbus data into your calculator.
   Enter the San Francisco data into your calculator.

2. Determine an appropriate window for the box and whisker plots.

3. Graph a box and whisker plot for both sets of data.

4. Determine the five-number summary for both sets of data by using the trace feature.

	Columbus, OH	San Francisco, CA
a. minimum		
b. lower quartile		
c. median		
d. upper quartile		
e. maximum		

Box and Whisker Plots

Try these problems using a graphing calculator. Find the measures of central tendency and variance for each problem, then create a box and whisker plot. You will need to record your responses on a separate sheet of paper or on the back.

1. Number of homeruns for each Oriole player (with 130 at bats or more) in 1999.

Cal Ripken	18	B.J. Surhoff	28	Will Clark	10
Albert Belle	37	Jeff Conine	13	Brady Anderson	24
Rich Amarall	0	Mike Bordick	10	Jerry Hairston	4
Delino Deshields	6	Charles Johnson	16	Jeff Rebolet	0

2. Coldest ever (on record) temperature for each month for Baltimore, MD.

Jan	¯7°F	Feb	¯3°F	Mar	6°F	Apr	20°F	May	32°F	June	40°F
July	50°F	Aug	45°F	Sept	35°F	Oct	25°F	Nov	13°F	Dec	0°F

3. Amount of rainfall for Honolulu, Hawaii each month in 1999.

Jan	3.4	Feb	2.6	Mar	2.8	Apr	1.3	May	1.0	June	0.4
July	0.6	Aug	0.6	Sept	0.7	Oct	2.0	Nov	2.6	Dec	3.5

4. All-Time Box Office Grosses (as of June 1999)

Titanic (1997)	600,788,188	Star Wars (1977)	460,998,007
E.T. (1982)	399,804,539	Jurassic Park (1993)	357,067,947
Forrest Gump (1994)	329,690,974	The Lion King (1994)	312,855,561
Return of the Jedi (1983)	309,161,884	Independence Day (1996)	306,169,255
The Empire Strikes Back (1980)	290,268,568	Home Alone (1990)	285,016,000

5. Age of each president at Inauguration

President	Age	**President**	Age	**President**	Age
Washington	57	Buchanan	65	Harding	55
J. Adams	61	Lincoln	52	Coolidge	51
Jefferson	57	A. Johnson	56	Hoover	54
Madison	57	Grant	46	F. D. Roosevelt	51
Monroe	58	Hayes	54	Truman	60
J. Q. Adams	57	Garfield	49	Eisenhower	62
Jackson	61	Arthur	50	Kennedy	43
Van Buren	54	Cleveland	47	L. B. Johnson	55
W. H. Harrison	68	B. Harrison	55	Nixon	56
Tyler	51	Cleveland	55	Ford	61
Polk	49	McKinley	54	Carter	52
Taylor	64	T. Roosevelt	42	Reagan	69
Fillmore	50	Taft	51	Bush	64
Pierce	48	Wilson	56	Clinton	46

How Much is a Handful?
Answer sheet

Answers are dependent upon class data. If possible, show the results of the calculator work on the overhead screen.

Box and Whisker Plots

Use Extremes, quartiles and median to check box and whisker plots. Outliers, if any, are listed below. You may choose not to introduce outliers here.

1.

Mean	13.8333…	Median	11.5	Mode	0 and 10
Lower Extreme	0	Upper Extreme	37	Range	37
Lower Quartile	5	Upper Quartile	21	Interquartile Range	16

Outlier: None

2.

Mean	21.333…	Median	22.5	Mode	none
Lower Extreme	-7	Upper Extreme	50	Range	57
Lower Quartile	3	Upper Quartile	37.5	Interquartile Range	34.5

Outlier: None

3.

Mean	1.791666…	Median	1.65	Mode	.6 and 2.6
Lower Extreme	0.4	Upper Extreme	3.5	Range	3.1
Lower Quartile	0.65	Upper Quartile	2.7	Interquartile Range	2.05

Outlier: None

4.

Mean	365182092.3	Median	321273268	Mode	None
Lower Extreme	285016000	Upper Extreme	600788188	Range	315772188
Lower Quartile	306169255	Upper Quartile	399804539	Interquartile Range	93635284

Outlier: 600,788,188

5.

Mean	54.8333…	Median	55	Mode	51, 54, 55, & 57
Lower Extreme	42	Upper Extreme	69	Range	27
Lower Quartile	51	Upper Quartile	58	Interquartile Range	7

Outlier: 69