Goal 3: Data Analysis & Probability

Designing Models to Simulate Actual Events Using a Random Number Generator on a Graphing Calculator

OBJECTIVE	CORE LEARNING GOALS
• The student will design models to simulate actual events using a random number generator on a calculator. • The student will estimate the likelihood of a particular outcome using results of simulations.	3.1.1 The student will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results. 3.1.3 The student will calculate theoretical probability or use simulations or statistical inferences from data to estimate the probability of an event.
DRILL If you roll a single die, what is the probability that: 1. you roll a 5? 2. you roll an odd number? *(1. 1/6 2. 3/6 or ½)*	MATERIALS Activities "Simulation", "Conducting Simulations", "Simulations on the Graphing Calculator", and "What's Your Sign? Designing the Simulation" Graphing calculators

CALCULATOR SKILLS

Use **randInt** to generate a random integer within a range specified by lower and upper integer bounds

Use **randBin** to generate a random integer from a specified binomial distribution **(OPTIONAL EXTENSION)**

ACTIVITIES

1. **Drill**. Have the students work individually to complete this drill. This is designed to be a quick check on the students' understanding of probability. If this drill is hard for students, more work on probability needs to be done before students are ready for simulations.

2. **Introducing Simulation -** Before giving students any handouts, give them **Problem 1**. Ask students to discuss this problem with their group to determine what they think the answer is or how one could go about solving this type of problem. Get the group responses and explain that simulations are often used to determine the answer to this type of problem. (Note: Students should do simulations with hands-on materials such as coins, number cubes, and/or random number tables before using the calculator.) Review the basic steps involved in a simulation and model those steps using **Problem 1**. Demonstrate the **randInt** feature of the calculator by showing the student what happens when you enter **randInt** (1, 10) and press enter several times. Students will see that the calculator generates a random interger between 1 and 10 each time enter is pressed. The next step is to add one more number to speed up the process. Type **randInt** (1, 10, 3) and then **randInt** (1, 10, 5) and ask the students to explain what is happening. They will see that the additional number allows you to generate and display more than one number at a time. The students are ready to explain the command **randInt**(1, 2, 7) and carry it out. Have pairs of students collect their results and share them with the class.

3. **Investigating Simulation** – Allow the students to work **Problem 2** and **Problem 3** with their groups. Discuss the results with the entire class and make sure that the students understand the calculator commands as well as how the simulations work.

4. **Additional problems –** "Conducting Simulations" & "Simulations on the Graphing Calculator"

ASSESSMENT	HOMEWORK
Summary Questions: *(Context may need to be explained.)* A goalie saves half of the shots on goal. Suppose there were twelve shots in a game. 1. Describe how you would conduct one trial of a simulation that models the results of the shots on goal. (*Using the digits 1 and 2, let 1 represent saving the shot and 2 represent missing the shot. Generate 12 of these digits to represent each of the twelve shots on goal.)* 2. Suppose the goalie saved 2/3 of the shots on goal. Describe how you would conduct one trial of a simulation that models the results of the shots on goal. *(Using the digits 1, 2, and 3, let 1 and 2 represent saving the shot and 3 represent missing the shot. Generate 12 of these digits to represent each of the twelve shots on goal.)* 3. How many trials should be conducted to obtain reasonable results? Use mathematics to justify your answer. *(One hundred trials should be reasonable for this situation. In practice, the more critical the situation, the more trials are preformed. For instance, more trials would need to be performed for medical research than for a taste test on soda preference.)*	"What's Your Sign? Designing the Simulation"

Simulation

Simulation is a modeling technique that is used to help people answer questions about real situations by using probability experiments that are similar to the real situation. It is important to understand the basic steps involved in conducting a simulation. A simple example is given to illustrate the process.

Problem 1: What are the chances that when you toss a coin 7 times you will have a run of at least 3 consecutive heads or 3 consecutive tails?

Step 1 State the Problem

If you toss a coin 7 times, what is the probability that you will have a run of at least 3 consecutive heads or 3 consecutive tails? (Note: It is important to have all of the information available and to understand the objective of the simulation.)

Step 2 State the Assumptions

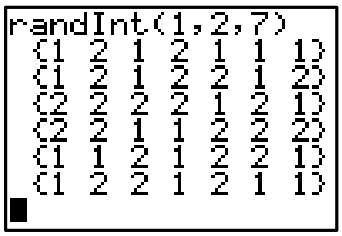
When you toss a coin, a head or a tail is equally likely to appear. Tosses are independent. In other words, the outcome of any toss will not influence the next toss.

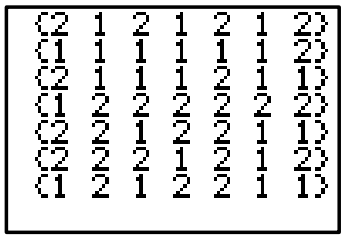
Step 3 Select a Model to Generate Outcomes

Let the digit 1 represent heads and the digit 2 represent tails. Use a random number generator or a random number table for getting digits.

Step 4 Simulate Many Repetitions

For this problem use the random integer feature of the TI-83. (Note: randInt is found by pressing MATH then PRB and selecting 5:randInt. The numbers 1, 2, 7 indicate that you are generating 7 random integers ranging from 1 to 2.)





Each time you press the enter key you generate another repetition of the experiment. When you are estimating the probability of getting a run of 3 heads or 3 tails when a coin is tossed 7 times, the more repetitions you have, the better your estimate will be. The results displayed on the calculator screen show that out of 13 trials there were 7 times when there were 3 heads or 3 tails in a row.

Problem 2: What are the chances that when you toss a coin 7 times you will get exactly 3 heads? Use the random binomial feature of the TI-83 to generate 5 trials of 7 tosses at a time. Then each time you press enter another 5 trials are simulated. (Note: randBin is found by pressing MATH then PRB and selecting 7:randBin. The numbers 7, .5, 5 indicate that you are completing 7 tosses with a probability of success (heads) of 0.5 and you are generating 5 trials at one time.)

```
randBin(7,.5,5)
      {3 0 5 3 3}
      {2 5 4 3 4}
      {5 5 2 5 5}
      {3 4 5 4 4}
      {3 4 4 6 4}
      {2 3 4 1 4}
```

Out of the 30 simulations generated by the calculator, exactly 3 heads appeared 7 times. So the experimental probability is (7/30) = 0.2333.

Problem 3: If you roll a die 5 times, what is the probability that you will get exactly one 6?

Method 1:

```
randInt(1,6,5)
      {4 3 5 3 6}
      {3 3 1 4 5}
      {4 4 3 2 3}
      {2 2 2 2 2}
      {5 3 4 4 2}
      {6 1 3 3 3}
```

```
      {1 4 4 1 2}
      {4 2 6 4 5}
      {3 4 4 3 6}
      {1 4 1 6 2}
      {1 6 5 2 6}
      {3 4 1 2 4}
      {5 3 6 4 6}
```

Since you rolled exactly one 6 in 5 out of 13 trials, the experimental probability is (5/13) = 0.3846.

Method 2:

```
randBin(5,(1/6),
5)
      {2 0 1 1 1}
      {1 1 0 1 2}
      {0 0 2 0 1}
      {2 2 0 1 0}
      {0 0 0 0 1}
```

Since you rolled exactly one 6 in 9 out of 25 trials, the experimental probability is (9/25) = 0.36.

Conducting Simulations

Describe how you would conduct one trial of a simulation model for each of the following situations using the graphing calculator.

1. Based on his history, Leon has an 80% chance of making a foul shot in a basketball game. Suppose Leon attempts 18 foul shots in a game. Describe one trial of a simulation model for Leon's foul shot results in a game.

2. Based on her history, Mindy scores on $\frac{3}{5}$ of her shots on goal in a field hockey game. Suppose she attempts 8 shots on goal in a game. Describe one trial of a simulation model for Mindy's shots on goal results in a game.

3. The *Bumble Bees'* chance of winning a football game is 20%. Suppose they play 15 football games in a season. Describe one trial of a simulation model for their 15 game season.

4. Carlos has two chances to get the correct answer on a multiple-choice question with three choices. Suppose he guesses. He will answer correctly on the first try $\frac{2}{3}$ of the time. If he has to try again, he has a 50% chance of getting the correct answer. Describe one trial of a simulation model for Carlos getting the correct answer on a multiple-choice question.

5. Based on his history, Anthony has a 77% chance of making a foul shot in a basketball game. Suppose he makes 16 shots in a game. Describe one trial of a simulation model for Anthony's foul shot results in a game.

Simulations on the Graphing Calculator

The Texas Tornadoes and the Montreal Marauders will play in next week's Hockey Championship. Based on regular season play, the Tornadoes have a 60% chance of beating the Marauders in any one game.

1. If the championship were decided in only 1 game, what is the probability that the Marauders will win the championship?

2. Suppose the two teams played 100 games. How many games would you expect each team to win? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

3. Describe how you would design one trial of a simulation model for a one-game series. Use the random number generator on the calculator. How will you define the numbers to represent a Texas or a Montreal win?

4. Is it appropriate to use a coin flip as a model to determine which team wins a game? Use mathematics to justify your answer.

Suppose the teams play a **five game series**.

5. How many games need to be won by a single team to win and end the series?

6. Describe how to conduct one trial of a simulation model for this series?

7. How could you use this model to estimate the probability that the Tornadoes will win the series?

8. Conduct 20 trials and record the results of each trial on a separate sheet of paper. Summarize your results by completing the frequency table below. Note: the series will end as soon as one team wins three games.

Series Winner	Frequency
Tornadoes	
Marauders	
Total	20

9. Based on your data, what is the probability that Texas wins the Hockey Championship?

What's Your Sign?
Designing the Simulation

There are twelve zodiacal signs. What is the probability that at least two people in a group of five have the same zodiacal sign?

1. a. How many possible outcomes are there for the zodiacal sign of each person?

   b. What is the probability of each outcome?

2. How could random numbers be used to model the possible outcomes?

3. Describe one simulation trial that shows the zodiacal signs for each person in a group of five people. How would you determine if two people in the group have the same zodiacal sign?

4. Conduct 20 trials, then share your results with others. Record your results in the frequency table below.

Frequency Table
What's Your Sign?

Are there *any* people with the same sign?	Frequency	
	Individual	Class
yes		
no		
Total # of Trials		

5. Based on your data, what is the probability that at least two people in a group of five have the same zodiacal sign? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

Conducting Simulations
Answer Key

1. You will need 10 digits, eight of which will represent making a foul shot, and the remaining two digits will represent missing the shot. Generate 18 digits at a time to represent the outcome of each attempted shot and observe the results.
2. You will need 5 digits, three of which will represent making a shot and the remaining two digits will represent missing the shot. Generate 8 digits at a time to represent the outcome of each attempted shot and observe the results.
3. You will need 10 digits, two of which will represent winning a football game and the remaining eight digits will represent losing a game. Generate 15 digits at a time to represent the outcome of each game in the season and observe the results.
4. You will need 6 digits (1, 2, 3, …, 6) to represent the responses to a question. Generate two digits at a time. Observe the first digit, if the first digit is a 1, 2, 3, or 4, then Carlos answers correctly on the first try. If the first digit is a 5 or 6, look at the next digit. If the next digit is a 1, 2, or 3, then Carlos answers correctly on the second try. If the second digit is 4, 5, or 6, then Carlos answers incorrectly.
5. You will need 100 digits, seventy-seven of these numbers will represent making a foul shot, and the remaining thirty-three digits will represent missing the shot. Generate 16 digits at a time to represent the outcome of each attempted shot and observe the results.

Simulations on the Graphing Calculator
Answer Key

1. 40%
2. Tornadoes should win approximately 60 games and Marauders should win 40 games.
3. Answers will vary. One possibility is to generate 10 numbers, let 6 of the numbers represent a Tornado win and let the other 4 represent a Marauder win.
4. Because the teams do not have an equal chance of winning (i.e. heads – Tornado win and tails – Marauder win), a coin can not be used to model the results of a single game.
5. 3 games
6. Using the number values assigned in #3, look at up to five digits at a time to observe who wins each game in the series. As soon as a team wins the series, begin a new trial.
7. Conduct many, many trials, and determine the probability that the Tornadoes will win.
8. Answers will vary. The results of one simulation are shown below:

Series Winner	Frequency
Tornadoes	16
Marauders	4
Total	20

9. Answers will vary. According to the above results: $\frac{16}{20} = 0.80$.

Designing the Simulation
Answer Key

1. a. 12

   b. $\frac{1}{12} = 0.083$

2. Use digits 1, 2, 3, …, 12 to represent the possible zodiac signs.
3. Generate these numbers (1 - 12) five at a time, and observe whether or not there are any matches.
4. Answers will vary. The results of twenty trials are shown below.

Frequency Table
What's Your Sign?

Are there *any* people with the same sign?	Frequency	
	Individual	Class
yes	12	
no	8	
Total # of Trials	20	

5. Answers will vary. According to the results above, $\frac{12}{20} = 0.60$.