Goal 3: Data Analysis & Probability

Designing Models to Simulate Actual Events Using a Table of Random Digits

OBJECTIVE	CORE LEARNING GOALS
• The student will design models to simulate actual events using a table of random digits. • The student will estimate the likelihood of a particular outcome using results of simulation.	3.1.1 The student will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results. 3.1.3 The student will calculate theoretical probability or use simulations or statistical inferences from data to estimate the probability of an event.

DRILL

A couple decides to have two children. They want to determine the probability of having two girls, so they simulate the births of their children by flipping 2 coins 50 times. The results of the simulation are shown in the table.

Possible Family	Frequency
2 boys	13
1 boy and 1 girl	26
2 girls	11
Total	50

Using the results of the simulation, estimate the likelihood of the couple having two girls.
(according to simulation: $11/50 = 0.22$ *or 22%)*

MATERIALS

Activities: "Using Random Digit Tables," "Guessing on a Multiple-Choice Test," and "Modeling Using a Table of Random Digits"
Answer Keys
Table of Random Digits

CALCULATOR SKILLS

none

ACTIVITIES

1. **Drill.** Students will estimate the likelihood of an event using results of a simulation.
2. **Exploration.** Students will investigate how to use a random digit table. They will conduct simulations using a random digit table. At first, teachers may want to instruct students to begin at the same point in the random digit table when conducting simulations. As students become more comfortable, they should begin at different points in the table.
3. **Class Discussion.** Students need to understand that selecting pairs of numbers at a time or skipping numbers in the table will not affect the "randomness" of their results. Teachers may want to discuss how to simulate events with outcomes that are more complex.
4. Additional problems. **"Modeling Using a Table of Random Digits"**

ASSESSMENT	HOMEWORK
Summary Questions: A girl has six sweaters that she can wear to school. Explain how you could use the table of random digits to determine whether it is likely that she will wear the same sweater more than once in a five-day week. Assume that each sweater selection is independent of the others. *(Use the digits 1-6 to represent the six sweaters, disregard the digits 0, 7, 8, and 9. Begin at a random point in the table and look at the following five numbers, ignoring any of the unused digits. These five numbers represent each day of the week. The number of 1s that appear will correspond to the number of times she wore the first sweater. The number of 2s that appear will correspond to the number of times she wore the second sweater, and so forth.)*	"Modeling Using a Table of Random Digits"

Using Random Digit Tables

Examine the following table of random digits.

62207	96845	33122	61147	93253	60200	85048	61922	37863	20812
91294	71538	27054	33696	24444	70998	51609	69031	36872	55220
12148	31711	35563	05855	53337	21329	52349	77117	73675	24707
79180	65454	79577	24794	73456	45187	81324	56924	58063	79666
24439	91645	83361	59133	13884	83711	95824	86031	69907	38546
04835	99007	56062	87960	01512	90025	93472	74303	05549	72506

There are 300 digits between 0 and 9 (inclusive) in the table.

1. About how many 5s would you expect to find? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation. Compare your answer to the actual number of 5s that appear.

2. About what percentage of digits in the table would you expect to be a 5? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

3. About what percentage of digits in any large table of random digits from 0 to 9 will be even? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

4. About what percentage of digits in any large table of random digits will be less than 5? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

5. About what percentage of digits in any large table of random digits will be either a 2 or a 5? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

6. About what percentage of the 1s in a large table of random digits will be followed by a 2? Use mathematics to justify your answer.

Some situations require the random assignment of people to different groups. For example, in many experiments scientists need to separate subjects into treatment and control groups. To maintain similarity in the two groups, scientists will often label subjects numerically, use a random device to select those numbers, and assign the corresponding subjects to groups. **For the following activities use the Table of Random Digits provided.**

7. A teacher must choose two students from a group of ten to participate in a certain activity. To avoid favoritism, she assigns numeric labels to each of her students as follows:

Label	Student	Label	Student
0	Amanda	5	Lynn
1	Bill	6	Malcom
2	Daniel	7	Neal
3	Emilio	8	Samantha
4	Jacob	9	Tracy

a. Beginning on line 2 of the Table of Random Digits, what are the first two digits?

b. Which *two* students would be chosen for the activity according to the labels the teacher assigned?

c. Suppose the teacher wanted to select *four* students from the group of students in the table. Which students would be selected if she selected digits beginning on line 4?

8. Consider a group of one hundred students.

a. How would you assign labels to these students?

b. Describe how you could use a table of random digits to select 5 students from this group of 100.

b. Begin on line 10 in the Table of Random Digits. Which five labels would you select?

9. Consider a group of twenty-six students.

a. How would you assign labels to these students?

b. Describe how you could use a table of random digits to select eight students from this group of 26.

c. Begin on line 20 in the Table of Random Digits. Which eight labels would you select?

Guessing on a Multiple-Choice Test

Suppose this week's history test consists of twenty multiple-choice questions. You did not study, so you will guess the correct response to each question.

1. If there are five responses from which to choose, only one of which is correct, what is the likelihood that you will answer any one question correctly? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

2. For this investigation we will use a table of random digits to simulate taking the history test. If there are ten digits (0, 1, 2, …, 9), then how many of these digits should represent a correct response? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

3. a. Which digits would you like to label as a correct response?

   b. Which digits would you like to label as an incorrect response?

The string of twenty digits below was taken from a table of random digits. Let each digit correspond to a question on the history test.

85392 41532 86687 16444

4. According to the digits above and the labels assigned in #3, how many questions were answered correctly?

5. Use the table of random digits provided to conduct a total of 20 trials for this simulation exercise. Record the total number of correct responses for each trial in the table below.

Trial #	# Correct	Trial #	# Correct	Trial #	# Correct	Trial #	# Correct
1		6		11		16	
2		7		12		17	
3		8		13		18	
4		9		14		19	
5		10		15		20	

6. a. You need at least 18 correct responses to receive an A on the test. Based on the results of your simulation, what is the probability of earning an A?

b. You need 16 or 17 correct responses to receive a B on the test. Based on the results of your simulation, what is the probability of earning a B?

c. You need 14 or 15 correct responses to receive a C on the test. Based on the results of your simulation, what is the probability of earning a C?

d. You need at least 12 correct responses to pass the test. Based on the results of your simulation, what is the probability of passing?

e. Based on the results of your simulation, what is the probability of failing the test?

f. What is the sum of your result from parts d and e? Will this sum always be the same? Use mathematics to justify your answers.

Table of Random Digits

Line									
1	89470	34231	77439	77008	03569	51225	26932	20265	76484
2	03404	42582	46841	77983	69237	45143	43982	33246	21696
3	50883	44636	05135	50051	13690	41800	58902	22068	49406
4	14805	51031	13002	98577	48318	59532	81291	30759	14972
5	50546	42082	21646	52428	74961	58761	11123	32750	07054
6	42966	25507	26774	13076	16409	93270	86194	77944	24690
7	14004	81839	28420	83942	67410	13910	80304	36011	12854
8	61425	10877	33037	24604	98278	77528	60105	01216	42502
9	31121	28143	72110	25381	54152	47658	94164	86893	14053
10	39391	28893	14835	77356	75610	96239	71092	81662	96176
11	13588	22830	48029	91989	56454	19372	27986	00404	30099
12	73615	78268	38308	20987	35019	70611	02795	04544	14743
13	07343	40442	85063	75853	69891	88510	58943	37011	15207
14	06762	58658	50386	08921	17347	18326	98143	04149	63934
15	16441	40374	47092	02066	54596	09259	77467	81449	94105
16	05688	54317	82513	03057	96689	94479	97207	17273	18225
17	80606	99756	55613	19083	77296	12819	85948	28337	20757
18	64121	61750	68206	36219	98527	23798	51801	52193	14432
19	53624	14443	39067	199777	77557	66765	76195	02985	74886
20	34356	66107	21627	12656	21678	94370	75697	62006	60762
21	60909	60720	30485	60729	46781	43236	05752	50464	27880
22	57512	31121	79024	45654	24580	16824	80279	66706	85793
23	82180	06973	07739	28815	13380	57084	35223	52327	59708
24	86766	72100	03589	76634	16786	14589	11059	29493	24922
25	39202	80559	22692	57183	73478	76522	90021	04754	98055

Modeling Using a Table of Random Digits

1. Describe how you could use random digits to model each of the following situations.

   a. Based on his history, Leon has an 80% chance of making a foul shot in a basketball game. Suppose Leon attempts 18 foul shots in a game.

   b. Mindy's coach has watched her all season and determines that she scores on $\frac{3}{5}$ of her shots on goal in a field hockey game. Suppose she attempts 8 shots on goal in a game.

   c. As goalie, Anthony saves $\frac{2}{3}$ of attempted shots in a lacrosse game. Suppose there are 12 attempted shots on goal in a game. (Hint: It is not necessary to use all ten digits. You can skip particular digits in the table, and the distribution of remaining digits is still random.)

   d. The *Bumble Bees'* chance of winning a football game is 45%. Suppose they play 15 football games in a season. (Hint: Consider using two digits at a time. The distribution 100 double-digits is still random.)

2. The Bears and the Lions will play in next week's Hockey Championship. Based on regular season play, the Bears have a 30% chance of winning each game. Suppose the teams play a five game series, in which one team needs to win 3 games to end the series and win the championship.

   a. Describe how to conduct one trial of a simulation model for this series using a table of random digits.

   b. How could you use this model to estimate the probability that the Bears will win the series?

c. Conduct 20 trials and record the results of each trial on a separate sheet of paper. Summarize your results by completing the frequency table below. Note: the series will end as soon as one team wins three games.

Series Winner	Frequency
Bears	
Lions	
Total	20

d. What is your estimate of the probability that the Bears win the hockey championship? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

3. Your softball team is to play its rival in a three-game series. The two teams are evenly matched. Suppose you want to estimate the likelihood of your team winning the series, in other words, winning at least two of the three games.

a. A possible model for the outcome of a single game is to flip a coin and let heads represent the event that your team wins. Is this a good model? Use mathematics to justify your answer.

b. Suppose the two teams are *not* evenly matched. Could you use a coin to represent the outcome of a single game in this situation? Use mathematics to justify your answer.

Using Random Digit Tables
Answer Key

1. Expected: $\frac{1}{10}(300) = 30$, since 10% of the 300 digits should be a five.
   Actual: 34, difference of 4
2. We'd expect that 10% of all the digits in any table of random digits would be fives.
3. Half of the digits in any large table of random digits should be even.
4. Half of the digits in any large table of random digits should be less than five (0, 1, 2, 3, 4).
5. About 20% of the digits in any large table of random digits should be either a 2 or a 5.
6. 10% of the 1s in a table of random digits should be followed by a 2. The random numbers are independent of each other.
7. a 0, 3
   b. Amanda, Emilio
   c. 1:Bill, 4:Jacob, 8:Samantha, 0:Amanda
8. a. One possible way to label these students would be to use 00, 01, 02, 03, …, 99.
   b. Pick a line in the table from which to start. Choose pairs of digits until five different pairs of labels have been chosen.
   c. 39, (skip the next 39), 12, 88, 93, 14
9. a. One possible way to label these students would be to use 00, 01, 02, …, 25 and skip all other pairs of digits.
   b. Pick a line in the table from which to start. Choose pairs of digits until eight different pairs of labels have been chosen.
   c. 07, 21, 26, 20, 06, 09, 03, 04 Skip unused labels and repeats.

Guessing on a Multiple-Choice Test
Answer Key

1. $\frac{1}{5} = 0.20$
2. Two digits should represent a correct response. (2/10) = (1/5)
3. a. Answers will vary. One possible way to label a correct response using 0 or 1.
   b. All other digits should represent an incorrect response. In this case: 2, 3, 4, …, 9.
4. Answers will vary. Using the above labels, only two responses are correct.
5. Answers will vary. Beginning on line 1 of the Table of Random Digits, the results follow:

Trial #	# Correct	Trial #	# Correct	Trial #	# Correct	Trial #	# Correct
1	4	6	10	11	6	16	7
2	3	7	5	12	3	17	4
3	3	8	7	13	6	18	7
4	1	9	4	14	5	19	8
5	1	10	3	15	6	20	2

6. a. Answers will vary. For the above results, 0.00.
   b. Answers will vary. For the above results, 0.00.
   c. Answers will vary. For the above results, 0.00.
   d. Answers will vary. For the above results, 0.00.
   e. Answers will vary. For the above results, 1.00.
   f. They add up to 1.00. The sum will always be 1 because passing and failing the test include all possible results of taking the test. Therefore, their probability must be 1.

Modeling Using a Table of Random Digits
Answer Key

1. a. Answers will vary. One possibility is to let digits 0, 1, 2, …, 7 represent making a foul shot and digits 8 and 9 represent missing one. Observe eighteen digits at a time.
   b. Answers will vary. One possibility is to let digits 0, 1, 2, …, 5 represent making a goal and digits 6, 7, 8 and 9 represent missing one. Observe eight digits at a time.
   c. Answers will vary. One possibility is to let digits 0 and 1 represent a save, let 2 represent a miss, and ignore digits 3, 4, 5,…, and 9. Observe digits (0, 1, 2) twelve at a time.
   d. Answers will vary. One possibility is to let digits 00, 01, 02, …, 44 represent making a win and digits 45, 46, 47, … 99 represent a loss. Look at fifteen double-digits at a time.
2. a. Answers will vary. One possibility is to let 0, 1, 2 represent a Bear win and 3, 4, 5, …, 9 represent a Lion win. Look at up to five digits at a time to observe who wins each game in the series.
   b. Conduct many, many trials, and determine the proportion of series wins by the Bears.
   c. Answers will vary. Beginning on Line 10 the results would follow:

Series Winner	Frequency
Bears	3
Lions	17
Total	20

   d. Answers will vary. According to the above results: $\frac{3}{20} = 0.15$.
3. a. Yes. Since your team's probability of winning is 50%, tossing a coin could be used to model the series.
   b. No. The teams are not evenly matched, so your team's probability of winning is *not* 50%. Tossing a coin could *not* be used to model the series.