Goal 3: Data Analysis & Probability

Designing Models to Simulate Actual Events

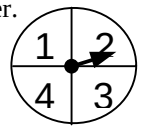
OBJECTIVE	CORE LEARNING GOALS
• The student will design models to simulate actual events using various random devices. • The student will estimate the likelihood of a particular outcome using results of simulation.	3.1.1 The student will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results. 3.1.3 The student will calculate theoretical probability or use simulations or statistical inferences from data to estimate the probability of an event.
DRILL Consider a well-shuffled standard deck of 52 cards. 1. What is the probability of drawing an Ace? $P(\text{Ace}) = 4/52 = 1/13$ 2. What is the probability of drawing the Jack of Spades? $P(\text{Jack of Spades}) = 1/52$ 3. What is the probability of drawing a face card (a Jack, Queen, King, or Ace)? $P(\text{face card}) = 16/52 = 4/13$ 4. What is the probability of drawing a red card? $P(\text{red card}) = 26/52 = 1/2$	MATERIALS Activities: "Guessing on a True/False Test," "Modeling with Other Random Devices," and "Simulating Families with Three Children" Answer Keys Coins, dice, spinners, or other similar devices

CALCULATOR SKILLS

none

ACTIVITIES

1. **Drill.** The drill provides a review of calculating theoretical probabilities.
2. **Exploration.** The first activity asks the question, "What is the probability of passing a test, for which you have not studied, by guessing?" This is certainly a question of interest to students. This activity is different from previous activities because students are no longer trying to make predictions about a population. It is known that one has a 50% chance of responding correctly to a true-false statement. It is known that there are ten statements on the test. What we don't know is the likelihood of responding correctly to at least 6 of these statements, thus passing the test. We can use a coin to model the correctness of each response, since there is a 50% chance of landing on either side. If we flip the coin ten times, we can model the results of one test. However, if we flip the coin another ten times (we call each set of ten flips a **trial**), we may get different results. So we flip the coin many, many times to estimate various frequencies in the long-run. The more trials we conduct, the closer our estimates will be to the theoretical probability. *An explanation of the context of saves and attempted shots on goal may be required when completing the activity "Modeling with Other Random Devices".*
3. **Class Discussion.** It is often difficult to calculate theoretical probabilities, so we continue conducting trials (this is similar to sampling) in an attempt to estimate these values. Theoretical probabilities deal with long-run behavior, and because results vary we need many trials/samples to model this behavior. It is often difficult and impractical to conduct experiments or surveys over and over again, so we rely on simulation techniques to model natural phenomenon quickly.
4. **Additional problems.** "Simulating Families with Three Children"

ASSESSMENT	HOMEWORK
Summary Questions: A goalie saves $\frac{2}{5}$ of the attempted shots on goal. Suppose there were eight attempted shots in a game. 1. Can you use a coin to simulate the attempted shots in one game? Use mathematics to justify your answer. *(No. There are not enough outcomes in coin tossing to model the shots on goal.)* 2. Can you use a six-sided number cube to simulate the attempted shots in one game? Use mathematics to justify your answer. *(Yes. For example, let 1 and 2 represent making a shot, let 3, 4, and 5 represent missing the shot, and toss the cube again if it lands on 6.)* 3. Can you use this spinner to simulate the attempted shots in a game? Use mathematics to justify your answer.  *(No. There are not enough outcomes on the spinner to model the shots on goal.)* 4. Can you use a standard deck of 52 cards to simulate the attempted shots in one game? Use mathematics to justify your answer. *(Yes. For example, let all 10's, Jacks, Queens, Kings, and Aces represent making a shot, ignore all red 2's, and let all other cards represent missing the shot. Obviously, using a deck of cards would not be very efficient.)*	"Simulating Families with Three Children"

Guessing on a True/False Test

You are about to take a unit test in your history class, but you have not studied or reviewed any of the material covered on the test. The test consists of ten statements, which you are to label as true or false. You decide to guess at each response.

Suppose you need at least six correct responses to pass the test, and you want to estimate the likelihood that you will pass. To determine the likelihood of a possible score theoretically, we would need to list every combination of outcomes. There are too many combinations to list for this example. Therefore, you will **simulate** guessing on the test by flipping a coin ten times. Each coin flip represents one statement on the test. Let heads represent a *success* (selecting the correct response) and tails represent a *failure* (answering incorrectly).

1. Toss a coin 10 times and record your results in the table below.

Trial	1	2	3	4	5	6	7	8	9	10
Response										

a. According to your results, how many questions were answered correctly?

b. Suppose you tossed the coin ten more times. Would you expect to see the same
results?

Let each set of ten tosses represent a **trial** in this investigation. To estimate the probability of passing the test, you need to determine the experimental probability that yields at least six successes, or correct responses.

2. Perform twenty trials to simulate taking the test 20 times and record the number of successes for each trial.

Trial	# Successes in ten tosses	Trial	# Successes in ten tosses	Trial	# Successes in ten tosses	Trial	# Successes in ten tosses
1		6		11		16	
2		7		12		17	
3		8		13		18	
4		9		14		19	
5		10		15		20	

3. Based on the data, what is the probability of guessing the correct response at least 6 times, passing the test? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

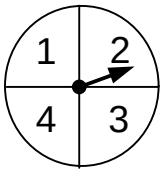
Modeling with Other Random Devices

1. A goalie saves half of the attempted shots on goal. Suppose there were twelve
attempted shots in a game.

   a. Describe how you could use a coin to simulate the attempted shots in one game.

   b. Describe how you could use a six-sided number cube to simulate the attempted shots in one game.

   c. Describe how you could use this spinner to simulate the attempted shots in a game.



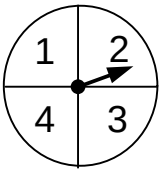
   d. Describe how you could use a standard deck of 52 cards to simulate the attempted shots in one game.

2. A basketball player makes 75% of her free-throw attempts. Suppose she attempts 20 free-throws in a game.

   a. Could you use a coin to simulate the attempted free-throws in a game?
   Use mathematics to justify your answer.

   b. Could you use a six-sided number cube to simulate the attempted shots in
   one game? Use mathematics to justify your answer.

   c. Could you use this spinner to simulate the attempted shots in a game? Use
   mathematics to justify your answer.



d. Could you use a standard deck of 52 cards to simulate the attempted shots
in one game? Use mathematics to justify your answer.

Simulating Families with Three Children

To simulate the births of three children to a family, a coin can be flipped three times and the results recorded. The results of 40 trials of this simulation are shown below. In each trial, H stands for heads, which represents the birth of a girl, and T stands for tails, which represents the birth of a boy.

HHT	TTT	HHH	HTT	HHH
THH	THT	HHH	HHH	HTH
TTT	THT	THH	THH	HHT
TTT	HTT	HTH	THH	TTH
HTT	HHT	THT	HTT	THT
THH	HHH	TTT	THT	HHT
TTH	HHT	HTH	TTT	HHT
THH	THT	HTT	THH	TTH

1. Conduct 10 more trials of this simulation. Record your results in the table below.

Trial #	1	2	3	4	5	6	7	8	9	10
Result										

2. Complete the following frequency table using the given 40 trials along with the results of your 10 trials in part a.

Type of Family	Frequency
No girls	
One girl and two boys	
Two girls and one boy	
Three girls	
Total Number of Trials	50

3. Use your frequency table to estimate the probability that a family with three children will have:

   a. Exactly one gir
   b. Exactly one boy
   c. Three girls
   d. Three boys

4. Do the four types of families in the frequency table appear to be equally likely? Use mathematics to justify your answer.

5. **Optional Extension Problem:** Based on your data, what is the mean number of girls in a family with three children? Use mathematics to explain how you determined your answer. Use words, symbol, or both in your explanation.

Guessing on a True/False Test
Answer Key

1. The results will vary. Consider the following ten tosses:

Trial #	1	2	3	4	5	6	7	8	9	10
Response	S	S	F	S	F	F	F	S	F	F

   a. Answers will vary. For the results above, there were 4 successes or correct responses.
   b. No.

2. The results will vary. Consider the following twenty trials:

Trial	# Successes	Trial	# Successes	Trial	# Successes	Trial	# Successes
1	3	6	2	11	4	16	4
2	6	7	4	12	6	17	5
3	7	8	4	13	3	18	3
4	6	9	7	14	5	19	7
5	6	10	7	15	2	20	5

3. Answers will vary. According to the above results, 8 out of 20 of the trials yielded at least six successes.

$$\frac{8}{20} = 0.40$$

Modeling with Other Random Devices
Answer Key

1. a. Answers will vary. Let heads represent saving a shot on goal and tails represent missing one. Flip the coin twelve times to observe the outcome of each attempt.
   b. Answers will vary. Let odd numbers represent saving a shot on goal and even numbers represent missing one. Toss the number cube twelve times to observe the outcome of each attempt.
   c. Answers will vary. Let odd numbers represent saving a shot on goal and even numbers represent missing one. Toss the number cube twelve times to observe the outcome of each attempt.
   d. Answers will vary. Let red cards represent saving a shot on goal and black cards represent missing one. Draw twelve cards from the deck (one card at a time, with replacement) to observe the outcome of each attempt.

2. a. No. A coin has two equally-likely outcomes. The outcomes, in this case hit or miss, are *not* equally-likely.
   b. Yes. Let ¾ of the numbers (for example 1, 2, 3, and 4) represent making a free-throw, and let the other numbers (for example 5 and 6) represent missing one. Toss the cube twenty times and observe results.
   c. Yes. Let ¾ of the numbers (for example 1, 2, and 3) represent making a free-throw, and let the other numbers (for example 4) represent missing one. Spin the spinner twenty times and observe the results.
   d. Yes. Let ¾ of the sits (for example hearts, spades, and diamonds) represent making a free-throw, and let the other suit (for example clubs) represent missing one. Draw twenty cards from the deck (one card at a time, with replacement) to observe the outcome of each attempt.

Simulating Families with Three Children
Answer Key

1. Answers will vary. Consider the following ten trials:

Trial #	1	2	3	4	5	6	7	8	9	10
Results	HHH	HHT	HTH	HTT	TTT	TTT	TTT	HTT	HTT	THH

2. Answers will vary. The frequency table with the results from the above trials is shown below:

Type of Family	Frequency
No girls	8
One girl and two boys	17
Two girls and one boy	19
Three girls	6
Total Number of Trials	50

3. a. Answers will vary. Using the above frequency table: $\frac{17}{50} = 0.34$

   b. Answers will vary. Using the above frequency table: $\frac{19}{50} = 0.38$

   c. Answers will vary. Using the above frequency table: $\frac{6}{50} = 0.12$

   d. Answers will vary. Using the above frequency table: $\frac{8}{50} = 0.16$

4. No. The four types of families are not equally likely because the frequencies would be different in the long run.

5. The theoretical expected value is 1.5 girls in a family of three children, since half of the children in a family should be girls. Using the above frequency table: $\frac{0(8)+1(17)+2(19)+3(6)}{50} = 1.46$.